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Civil Engineering

Soil Mechanics and Foundation Engineering

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Soil Mechanics and Foundation Engineering

Edition: 2018

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CHAPTER 01

Soil Formation

1.1 Definition of Soil

Generally, the term 'soil', is used for upper layer of the earth surface which can support plants.

For engineering purposes, soil is defined as the uncemented aggregate of mineral grains and decayed organic matter (solid particles) with liquid and gas in the empty spaces between the solid particles.

Soil is an unconsolidated material that has resulted from the disintegration of rocks by various weathering agencies like water, air etc. The soil may contain inorganic or organic matter and can be represented as a three phase system containing solids, water and air. Soil is used as a construction material in various civil engineering projects and it supports structural foundations.

1.2 Definition of Soil Mechanics

Soil mechanics is the branch of science that deals with the study of physical properties of soil and the behaviour of soil masses subjected to various types of forces.

The term 'Soil Mechanics' was given by the Dr. Karl Terzaghi in 1925, who is also known as Father of Soil Mechanics. According to Terzaghi, "Soil mechanics is the branch of civil engineering that concerns the application of the principles of mechanics, hydraulics and chemistry to engineering problems related to the soil."

1.3 Definition of Soil Engineering

Soil Engineering is the application of principles of soil mechanics to practical purposes. It is a broader term including soil mechanics, geology, structural engineering and soil dynamics which are essential to obtain practical solution to problems related to the soil. It includes site investigation, lab testing, design, construction and maintenance of foundations and earth retaining structures.

1.4 Definition of Rock Mechanics

A natural aggregate of mineral particles bounded by strong and permanent cohesive force is called 'rock'. Rock Mechanics is the science dealing with the application of the principles of mechanics to understand the behaviour of rock masses.

1.5 Geotechnical Engineering

Geotechnical Engineering is the sub-discipline of civil engineering that involves natural materials found closer to the surface of the earth. It includes the application of the principles of soil mechanics and rock mechanics to the design of foundations, retaining structure and earth structure. Geotechnical Engineering is a broader term which includes soil mechanics, rock mechanics, rock engineering, Geology and Soil Engineering.

1.6 Origin of Soil

Almost all the soils are formed by the disintegration of rocks either through physical or chemical weathering. If weathered sediments remain over parent rock, then soil is called 'Residual soil' and weathered sediments transported and deposited at some other place are called 'Transported soil'.

The process of soil formation is called 'Pedogenesis'.

The soil formation is cyclic which is called 'Geological cycle'.

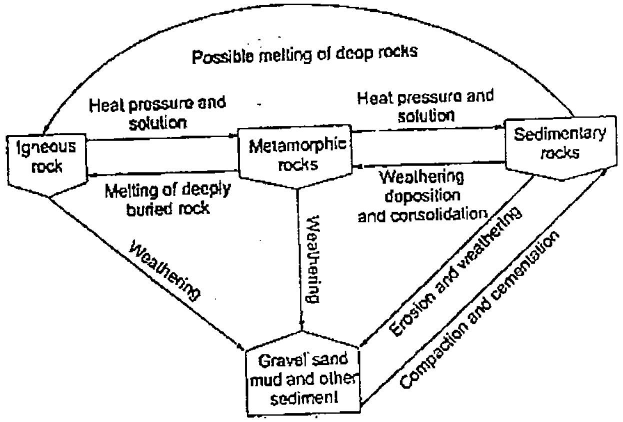


Fig. 1.1 Geological cycle (suggested by Bowles, 1984)

These are the stages in the geological cycle of soil formation in transported soil:

(1) weathering
(2) transportation
(3) deposition of weathered materials
(4) upheaval

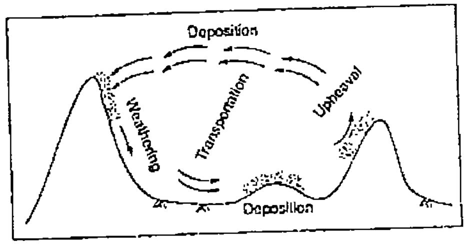


Fig. 1.2 Stages of Geological Cycle in case of transported soil

1.6.1 Weathering Stage

Rock disintegration, also called weathering, is one of the important geological process. Weathering may be physical (mechanical) or chemical.

Physical Weathering: In Physical weathering, rock disintegrates into smaller fragments due to various agencies like running water, heavy wind, temperature changes, rainfall, expansion due to freezing of water and human activities. Generally, sand and gravel fall in the category.

The mineral constituents remains same as that of parent rock, if soil is formed as a result of physical weathering.

Chemical Weathering: Fragmented rock materials obtained by physical weathering sometimes changes their mineral composition and new compounds are formed. This phenomenon is referred as 'Chemical Weathering'.

Chemical weathering is caused mainly by oxidation, hydration, carbonation, leaching water and organic acids.

Generally, clays and to some extent silts fall in this category.

1.6.2 Transportation and Deposition

The fragmented rock material obtained by weathering are transported by agents like running water (i.e. rivers), moving ice (i.e. glaciers) and blowing wind to new locations. The weathered fragmented materials transported and deposited at some other place are called 'Transported soil'.

On the basis of transporting agency, soil may be classified as:

Water transported soil: These sedimentary deposits are of three types

(i) Alluvial deposit
(ii) Lacustrine deposit
(iii) Marine deposit

(i) **Alluvial deposit (Alternate Layer of Sand + Silt + Clay):** Swift running water carry a large quantity of soil either in suspension or by rolling and sliding along the bottom of stream. When decrease in water velocity occurs, these sediments get deposited from suspension in running water. This type of soil is called 'Alluvial soil'.

Some of the examples of alluvial soil deposit are Alluvial cones, Natural levees and Delta

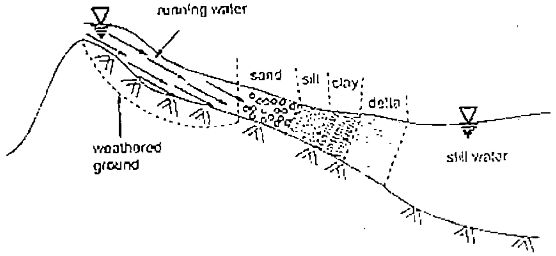


Fig. 1.3 Alluvial deposit

(ii) **Lacustrine deposit:** Soil carried by river, while entering a lake, deposits all the coarse particles because of a sudden decrease in velocity but the fine grained particles moves to the centre of the lake and settles when the water becomes still. Alternate layers are formed with season, and such lake deposits are called 'Lacustrine deposit'.

(iii) **Marine Soil:** These are soils that have been formed by sedimentation of soil particles in the deep sea water. The marine deposits have very low shearing strength and are highly compressible. They contain a large amount of organic matter. The marine clays are soft and highly plastic.

Wind Transported Soil: Like water, wind can erode, transport and deposit fine grain soils. The soils that have been transported and deposited by wind are called 'Aeolian deposits'. Dunes are formed due to accumulation of such wind deposited sand.

Loess: Loess is a silt deposit made by wind. These deposits have low density, slight cementation, high compressibility and poor bearing capacity when wet. The cementing is due to the presence of deposits of calcium carbonate, derived from decayed vegetable matter. When partially saturated, loess undergoes reduction in volume. This phenomenon, which occurs with increase in moisture content, without any charge in applied stress is termed as collapse compression, due to collapse of soil structure. Soils having susceptibility to collapse are called collapsible soils or metastable soils.

Glacial Deposits: Glaciers are large masses of ice formed by the compaction of snow. A glacier moves extremely slow but deforms and scour the surface and the bedrock over which it passes. Melting of a glacier cause deposition of all the materials and such a deposit is referred to as till. Drift is general term used for the deposits made by glaciers directly or indirectly.

The particles found in glacial deposits are generally angular, in contrast to the more rounded particles associated with typical waterborne deposits.

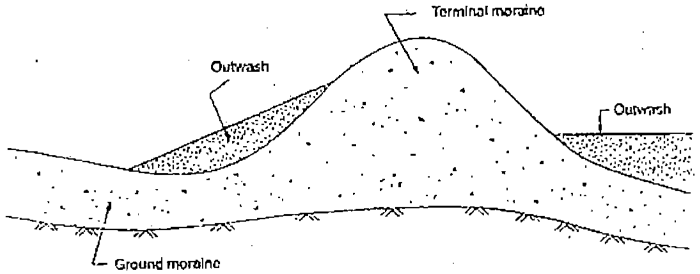


Fig. 1.4 Glacier Deposited Soil

Gravity Deposits: Gravity can transport materials for short distance. Gravity deposits are termed as colluvial soil or talus. These soils are found in mountain valley and formed by gravity force. On sloppy mountain due to moisture variation sediments start creeping down. Also due to swelling and shrinkage land slide may occur which may result into deposits in the valley. These soil consists of irregular particles of varying size.

Swamp and Marsh Deposits: In water stagnated areas where the water table is fluctuating and vegetational growth is possible, swamp and marsh deposits develop. These soils are soft, high in organic content and unpleasant in odour. Accumulation of partially or fully decomposed aquatic plants in swamps or marshes is termed 'muck or peat'. Muck soil is light in weight, highly compressible so this is not suitable for construction purpose.

1.7 Soil Deposits in India

India has been divided into five major soil groups

- Alluvial deposits
- Laterite soil
- Marine Deposits
- Black cotton soil
- Desert soil

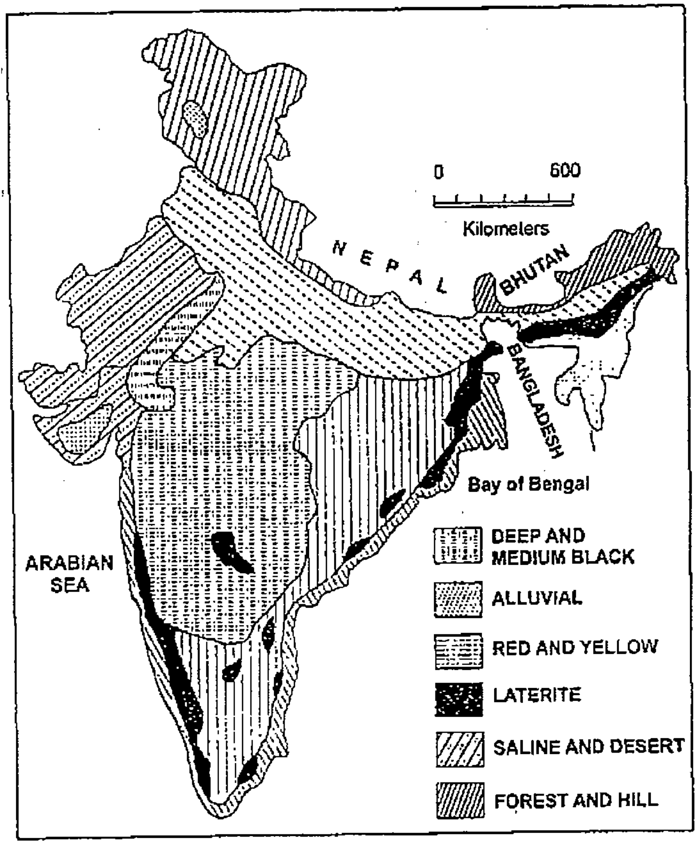


Fig. 1.5 Soil map of India

1.8 Organic and Inorganic Soils

Soils can also be classified as organic or inorganic soil.

- Organic soil are formed by growth and subsequent decomposition of plant. For example peat and mosses.
- In general, organic soil is that transported soil obtained from rock weathering which contains decomposed vegetable matter.
- Inorganic soils referred to ordinary soil obtain from rock disintegration due to weathering.

1.9 Common Types of Soils

- Loess: These are wind blown uniformly graded fine soil. Loess is formed in arid and semi-arid regions. Its colour is yellowish brown and deposits of this soil are found in Rajasthan and North Gujarat.

- **Caliche:** It is cemented soil rich in calcium carbonate consisting of gravel, sand and clays. These are also wind blown in semi-arid climate and later on cemented by the calcium carbonate left out from the evaporation of capillary water.
- **Loam:** It is a mixture of sand, silt and clay in definite proportion which in some cases may consist of organic matter.
- **Cumulose:** Peaty (organic) soils are also called cumulose soil or muck. These are formed due to accumulation of organic content under waterlogged condition. It is generally found in the areas having deficient sewerage facilities or found after overflooding of the rivers.
- **Gumbo:** These are highly sticky, plastic and dark coloured soil.
- **Marl:** These are fine grained calcium carbonated soil of marine origin. These are formed due to decomposition of cell mass and bones of aquatic life.
- **Humus:** Humus is a mixture of mud and dead plants. The tiny pieces of rock and humus joint to make various soils.
- **Peat:** It is highly organic soil containing almost decomposed vegetable matter.
- **Gravel:** Gravel is a type of coarse grain soil having particle size in the range of 4.75 mm to 80 mm.
- **Sand:** They are cohesionless aggregates of rounded, sub angular or angular sediment in the range of 0.075 mm to 4.75 mm.
- **Silt:** It is a fine-grained soil, with particle size between 0.002 mm and 0.075 mm.
- **Clay:** It is an aggregate of mineral particles of microscopic and submicroscopic range. Clay may be organic or inorganic.
- **Cobbles:** Cobbles are large size particles in the range of 80 mm to 300 mm.
- **Boulders:** Boulders are rock fragment of large size (more than 300 mm).
- **Tuff:** These are small grained slightly cemented volcanic ash that has been transported by wind or water.
- **Bentonite:** It is a clay formed by chemical weathering of volcanic ash which have high content of montmorillonite. Pulverized slurry of bentonite is highly plastic and is often used as a lubricant in drilling.
- **Kaolin (China Clay):** It is very pure form of white clay, which is extensively used in ceramic industry.
- **Hardpans:** Hardpans are types of soils that offer great resistance to the penetration of drilling tools during soil exploration. These are generally dense, well graded, cohesive aggregates of mineral particles.
- **Varved Clays:** These are sedimentary deposits consisting of alternate thin layer of silt and clay. These clays are the result of deposition in lakes during periods of alternate high and low waters.
- **Till:** It is formed by glaciers and iceberg and may contain mixture of gravel, sand, silt and clay. These soils are well graded.

Summary

- Soil is an unconsolidated material which consists of sediments derived from rocks.
- The soil formation process is cyclic, which is called 'Geological Cycle'.
- The process of soil formation is called 'Pedogenesis'.
- If weathered sediments remain over parent rock, then soil is called 'residual soil'.
- Weathered sediments transferred and deposited at other place are called transported soil.
- Major soil deposits of India are alluvial deposits, black cotton soil, laterite and lateritic soil, desert soil and marine deposits.

Objective Brain Teasers

Q.1 Glaciers are formed by
(a) Compaction and recrystallization of snow
(b) Continuous freezing of water
(c) A sudden drop in temperature below 0°C
(d) None of above process

Q.2 When the product of rock weathering are not transported as sediment but remain in place, then the soil is known as
(a) Alluvial soil (b) Glacial soil
(c) Residual soil (d) Aeolian soil

Q.3 Identify the true statements from the following
(a) A soil transported by gravitational force is called talus
(b) Lateritic soil is a category of organic soil
(c) Water held firmly to the clay particles has the same properties as ordinary water.
(d) A clay deposit which exibits no evidence of fissuring is described as intact.

Q.4 Bentonite is a material obtained due to the weathering of
(a) Lime stone (b) Quartzite
(c) Volcanic ash (d) Shales

Q.5 Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I	List-II
A. Wind	1. Lacustrine
B. Lake	2. Loess
C. Gravity	3. Till
D. Glacier	4. Talus

Codes:

	1	2	3	4
(a)	A	C	B	D
(b)	B	A	D	C
(c)	C	A	C	D
(d)	D	B	A	C

Q.6 Match List-I (Type of soil) with List-II (Mode of transportation and deposition) and select the correct answer using the codes given below the lists:

List-I
A. Residual soil
B. Loess
C. Peat
D. Varved clays

List-II
1. Soil transported by wind
2. Organic soil
3. Deposition in lake during periods of alternate high and low waters
4. Soil left in place after weathering of parent rock

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	2	3	4	1
(c)	4	1	2	3
(d)	3	2	1	4

Q.7 Which of the following type of soil is transported by gravitational force?
(a) Loess (b) Talus
(c) Drift (d) Dune Sand

Q.8 Lacustrine soil are soils
(a) transported by rivers and streams
(b) transported by glaciers
(c) deposits in sea beds
(d) deposited in lake beds

Q.9 Which one of the following soils is the Aeolian?
(a) Volcanic soil
(b) Residual soil
(c) Weathered soil
(d) Transported soil

Q.10 Assertion (A): Black cotton soils are clay and exhibit characteristic property of swelling.
Reason (R): These clays contain Montmorillonite which attracts external water into its lattice structure.
Codes:
(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Answers

1. (a) 2. (c) 3. (a) 4. (c) 5. (b)
6. (c) 7. (b) 8. (d) 9. (d) 10. (a)

Hints and Explanations:

Ans.7 (b)

Soil carried and deposited by river water are alluvial soils. Deposits made in lakes are lacustrine deposits. Wind transported soils are aeoline deposits. Drifts are made by glaciers. Deposits directly made by melting of glaciers are called till. Talus are colluvial soil deposits, transported by gravity.

■■■■

CHAPTER 02

Properties of Soils

2.1 Introduction

Soil is essentially made up of solid particles, with spaces or voids in between. The assemblage of particles in contact is usually referred to as the 'soil matrix' or the 'soil skeleton'. The intermittent void spaces are filled up by either air or water or both air and water. This means that an element of 'soil' may be considered as a three-phase material, comprising some solid (soil grains), some liquid (pore water) and some gas (pore air).

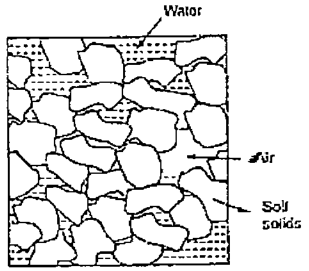


Fig. 2.1 Three phase solid-water-air system

2.2 Phase Diagram

- Soil mass, in general is a three phase system composed of solid, liquid and gaseous phase.
- Different phases present in soil mass cannot be separated. For better understanding, all three constituents are assumed to occupy separate spaces as shown in figure below.

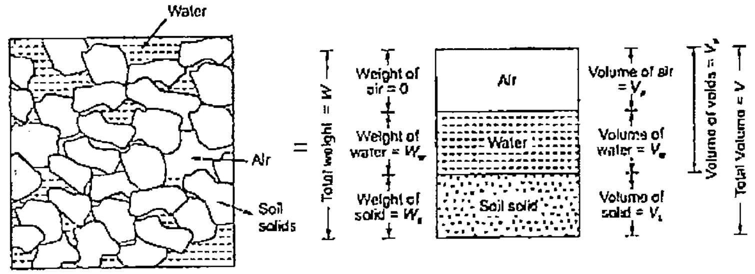
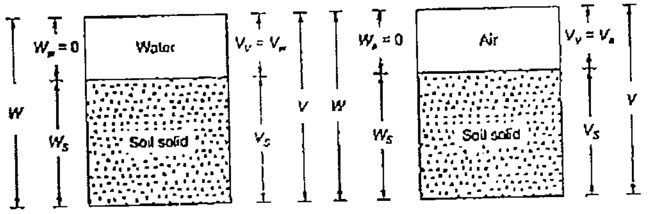


Fig. 2.2 Three Phase diagram.

- The diagrammatic representation of the different phases in a soil mass is called the 'phase diagram', or 'block diagram'.
- A three-phase diagram is applicable for a partially saturated soil ($0 < S < 1$)
- When all the voids are filled with water, the sample becomes saturated and thus the gaseous phase

is absent; whereas, in oven dry soil sample the liquid phase is absent. Thus, in saturated and oven dry soils, the three phase system reduces to two phase system.



(a) Water and soil solids (Fully saturated soil, S = 1)

(b) Air and soil solids (Oven dry soil, S = 0)

Fig. 2.3 Two Phase diagram

2.3 Basic Definitions

(i) Water Content (*w*)

- Water content (*w*) is also called moisture content. It is the ratio of weight of waterto the weight of soil solids.

$$w = \frac{W_w}{W_s}; \quad w \geq 0$$

- This is represented as a percentage.
- The water content of a oven dry soil is zero but natural water content for most soils is around 60%.
- There is no upper limit for water content. It can be greater than 100%.

NOTE

1. Fine-grained soils have higher values of natural moisture content as compared to coarse-grained soils.
2. There are four possible forms of water present in soil:
   (i) Gravity water (free water): Added duo to rain or flooding
   (ii) Capillary water : Extracted through capillary action
   (iii) Hygroscopic water : Water absorbed by oven dried sample when it is placed in open atmosphere
   (iv) Structural water : Water bounded in crystalline structure of soil
   On oven drying, gravity water, capillary water and hygroscopic water are removed but structural water remains present in soil mass.
3. Water content in soil represents gravity water, capillary water and hygroscopic water, which can be removed on oven drying.

(ii) Degree of Saturation(S)

- Degree of saturation (*S*) of a soil is defined as the ratio of the volume of water to the volume of voids in the soil mass.

$$S = \frac{V_w}{V_v} \times 100$$

---

where,
V_w = volume of water
V_v = volume of voids

- It is expressed in percentage.
- For dry soil, S = 0% and for fully saturated soil S = 100%, whereas partially saturated soil have $0 < S < 100\%$.

NOTE

- If soil is partially saturated, then total volume of soil and volume of void remain constant during variation of moisture content. If soil is super saturated due to further addition of water, then volume of void and total volume increases. Hence void ratio will change but degree of saturation remains constant equal to 100%.

(iii) Void Ratio (*e*)

- The void ratio (*e*) of a soil is defined as the ratio of the total volume of voids to the volume of solids.

$$e = \frac{V_v}{V_s}; \quad e > 0$$

where,
V_v = volume of voids
V_s = volume of soil solids

- It is expressed in decimal.
- In general $e > 0$, i.e. no upper limit for void ratio.
- Void ratio of fine grained soils are generally higher than those of coarse grained soils. The individual void spaces in coarse grained soil are larger than fine grained soils; but the tot al void space is generally more in fine grained soils.

(iv) Porosity (*n*)

- The porosity (*n*) of a soil is defined as the ratio of volume of voids to the total volume of soil.

$$n = \frac{V_v}{V} \times 100\%$$

where,
V_v = volume of voids
V = Total volume of soil

- It is expressed in percentage.
- In porosity, total volume of soil is used which includes volume of voids.
- Hence porosity (*n*) of soil cannot exceed 100%.
- The range of porosity is $0 < n < 100\%$.

NOTE

Void ratio (*e*) and porosity (*n*) both have same significance but void ratio (*e*) is more widely adopted than porosity because volume of solid which is used in void ratio is more stable tha n total volume used in porosity.

(v) Air Content (a_c)

- It is defined as the ratio of the volume of air to the total volume of voids present in soil.

$$a_c = \frac{V_a}{V_v}$$

where, V_a = volume of air in voids
V_v = volume of voids

- It is expressed in percentage.

(vi) Percentage Air Voids (n_a)

- Percentage air voids (n_a) is defined as the ratio of volume of air to the total volume of soil mass.

$$n_a = \frac{V_a}{V} \times 100$$

where, V_a = volume of air
V = total volume of soil

- It is expressed in percentage.

(vii) Unit Weights

(a) Bulk Unit Weight (γ_t)

- It is the ratio of total weight of soil to the total volume of soil mass.

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_w + V_s}$$

- It is expressed as $\frac{kN}{m^3}$ or $\frac{kgf}{cm^3}$

NOTE

- Bulk density is defined as the ratio of total soil mass to the total volume.

$$\rho_t = \frac{M}{V} = \frac{M_s + M_w}{V_s + V_w + V_s}$$

- It is expressed as $\frac{kg}{m^3}$

(b) Dry Unit Weight (γ_d)

- It is the ratio of total dry weight of soil to the total volume of soil mass.

$$\gamma_d = \frac{\text{Dry weight of soil}}{\text{Total volume}} = \frac{W_{dry}}{V}$$

- Dry unit weight is used as a measure of denseness of soil. More dry unit weight means more dense or compacted is the soil.

NOTE

- Dry density is defined as the ratio of total dry mass to the total volume.

$$\rho_d = \frac{M}{V} = \frac{M_{dry}}{V}$$

(c) Saturated Unit Weight (γ_{sat})

- It is defined as the ratio of total saturated weight of soil to the total volume of soil mass

$$\gamma_{sat} = \frac{W_{sat}}{V}$$

Do You Know?

- Saturated density is defined as the ratio of total saturated soil mass to the total volume of soil mass.

$$\rho_{sat} = \frac{M_{sat}}{V}$$

(d) Submerged Unit Weight or Buoyant Unit Weight (γ'_{sub} or γ')

- It is the ratio of buoyant weight of soil to the total volume of soil mass.

$$\gamma' = \frac{W_{sub}}{V}$$

- When soil is below water i.e. in submerged condition, a buoyant force acts on the soil solids which is equal in magnitude to the weight of water displaced by the soil solids. Hence the net weight of soil is reduced and reduced weight is known as buoyant weight or submerged weight.

$$\therefore \quad \gamma' = \frac{W_{sub}}{V} = \gamma_{sat} - \gamma_w$$

- γ' is roughly $\frac{1}{2}$ of saturated unit weight (γ_{sat})

Note: Submerged density or buoyant density, $\rho' = \rho_{sat} - \rho_w$

(e) Unit Weight of Water (γ_w)

- It is the ratio of weight of water to the volume occupied by the water

$$\gamma_w = \frac{W_w}{V_w}$$

- Unit weight of water depends on its temperature. However, the unit weight of water is taken to be constant as 9.81 kN/m³ or 1g/cc.
- It is expressed in $\frac{kN}{m^3}$ or $\frac{kgf}{cm^3}$

(f) Unit Weight of Solids (γ_s)

- It is the ratio of weight of soil solids to the volume occupied by the soil solids.

$$\gamma_s = \frac{W_s}{V_s}$$

- It is expressed in $\frac{kN}{m^3}$ or $\frac{kgf}{cm^3}$

(viii) Specific Gravity (G)

- Specific gravity of soil solids (G) is the ratio of the weight of a given volume of solids to the weight of an equivalent volume of water at 4°C.

$$G = \frac{W_s}{V_s \gamma_w} = \frac{\gamma_s}{\gamma_w} \qquad \left[\because \gamma_s = \frac{W_s}{V_s}\right]$$

- The specific gravity of most of the inorganic soils lies in the range of 2.65 to 2.80.
- For organics soils, it lies in the range of 1.2 to 1.40.

(ix) Apparent or Mass Specific Gravity (G_m)

- Mass specific gravity is defined as the ratio of the total weight of a given volume of soil to an equivalent volume of water.
- Mass specific gravity can be defined as the ratio of bulk unit weight of soil to unit weight of water.

$$G_m = \frac{W_t}{V\gamma_w} = \frac{\gamma_t}{\gamma_w}$$

If soil is in saturated state,

$$G_m = \frac{\gamma_{sat}}{\gamma_w}$$

If soil is in dry state,

$$G_m = \frac{\gamma_d}{\gamma_w}$$

Do You Know

- Generally, specific gravity is represented either at 27°C or at 20°C. If test temperature is different than the standard temperature, then correction has to be done as follows–

$$G = \frac{\gamma_s}{\gamma_w}$$

$\therefore$ $G \times \gamma_w = \text{constant}$

$\Rightarrow$ $G_{27^\circ C} \times \gamma_{w27^\circ C} = G_{T^\circ C} \times \gamma_{wT^\circ C}$

$\therefore$ $G_{27^\circ C} = G_{T^\circ C} \times \frac{\gamma_{wT^\circ C}}{\gamma_{w27^\circ C}}$

2.4 Some Important Relationships

1. Relation between W_s, W_w and W

From block diagram,

$$W = W_s + W_w + W_a$$

$$W = W_s + W_w + 0 \quad (\because W_a = 0)$$

$$W = W_s\left(1 + \frac{W_w}{W_s}\right)$$

$$W = W_s(1 + w) \quad \left(\because \text{Water content, } w = \frac{W_w}{W_s}\right)$$

$$\therefore \quad W_s = \frac{W}{1+w}$$

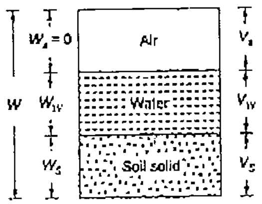


Fig.2.4

2. Relation between e and n

We know, Porosity, $n = \frac{V_v}{V} = \frac{V_v}{V_s + V_v} = \frac{\left(\frac{V_v}{V_s}\right)}{1 + \left(\frac{V_v}{V_s}\right)}$

$$\Rightarrow \quad n = \frac{e}{1+e} \quad \left(\because e = \frac{V_v}{V_s}\right)$$

3. Relation between e, S, w and G

We know,

$$\text{Void ratio, } e = \frac{V_v}{V_s}$$

Also,

$$e = \frac{V_v}{V_s} = \frac{V_v}{V_w} \times \frac{V_w}{V_s} = \frac{V_v}{V_w} \times \frac{W_w/\gamma_w}{W_s/\gamma_s} = \frac{V_v}{V_w} \cdot \frac{W_w}{W_s} \cdot \frac{G_s\gamma_w}{\gamma_w} = \frac{1}{S}wG_s$$

$$e = \frac{wG_s}{S}$$

or

$$Se = wG$$

4. Relation between γ_t, G_s, e, w and γ_w

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_v} = \frac{W_s\left(1 + \frac{W_w}{W_s}\right)}{V_s\left(1 + \frac{V_v}{V_s}\right)}$$

But $\frac{W_w}{W_s} = w$ and $\frac{W_s}{V_s} = \gamma_s = G_s\gamma_w$

$$\therefore \quad \gamma_t = \frac{G_s\gamma_w(1+w)}{1+e}$$

But $w = \frac{Se}{G_s}$

$$\therefore \quad \gamma_t = \left(\frac{G_s + Se}{1+e}\right)\gamma_w$$

Special Case (a): If soil is saturated, then

$\gamma_t = \gamma_{sat}$ and $S = 1$

Hence $\gamma_{sat} = \left(\frac{G_s + 1 \times e}{1+e}\right)\gamma_w$

or $\gamma_{sat} = \left(\frac{G_s + e}{1+e}\right)\gamma_w$

Special Case (b): If soil is dry, then

$\gamma_t = \gamma_d$ and $s = 0$

Hence $\gamma_d = \left(\frac{G_s + 0 \times e}{1+e}\right)\gamma_w$

or $\gamma_d = \frac{G_s\gamma_w}{1+e}$

Special Case (c): If soil is submerged, then

$$\gamma' = \gamma_{sat} - \gamma_w = \left(\frac{G_s + e}{1+e}\right)\gamma_w - \gamma_w$$

$$\gamma' = \left(\frac{G_s - 1}{1+e}\right)\gamma_w$$

5. **Relation between γ_t, γ_d, w**

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V}$$

$$\gamma_t = \frac{W_s(1 + W_w / W_s)}{V}$$

or $$\gamma_d = \frac{\gamma_t}{1+w} \qquad \left(\because \gamma_d = \frac{W_s}{V}\right)$$

6. **Relation between γ_d, G_s, w and n_a**

$$V = V_s + V_w + V_a$$

$$1 = \frac{V_s}{V} + \frac{V_w}{V} + \frac{V_a}{V} = \frac{V_s}{V} + \frac{V_w}{V} + n_a$$

or $$1 - n_a = \frac{V_s}{V} + \frac{V_w}{V} = \frac{W_s / G_s\gamma_w}{V} + \frac{wW_s / \gamma_w}{V} \qquad \left(\because V_w = \frac{wW_s}{\gamma_w}\right)$$

$$= \frac{\gamma_d}{G_s\gamma_w} + \frac{wW_s / \gamma_w}{V} = \frac{\gamma_d}{G_s\gamma_w} + \frac{w\gamma_d}{\gamma_w} = \frac{\gamma_d}{\gamma_w}\left(W + \frac{1}{G_s}\right)$$

or $$\gamma_d = \frac{(1-n_a)G_s\gamma_w}{1+wG}$$

Special Case (*a*): When $n_a = 0$, then soil become fully saturated at a given water content

Hence $$\gamma_d = \frac{G_s\gamma_w}{1+wG}$$

or $$\gamma_{sat} = \left(\frac{G_s + e}{1+e}\right)\gamma_w$$

7. **Relation between S, w, G_s, γ_t and γ_w**

$$\gamma_t = \left(\frac{G_s + Se}{1+e}\right)\gamma_w$$

$$\frac{\gamma_t}{\gamma_w} = \left(\frac{G_s + Se}{1+e}\right) = \left(\frac{G_s + wG_s}{1 + \frac{wG_s}{S}}\right)$$

$$\left(1 + \frac{wG_s}{S}\right) = \frac{G_s\gamma_w}{\gamma_t}(1+w)$$

$$\frac{1}{G_s}\left(1 + \frac{wGs}{S}\right) = \frac{\gamma_w}{\gamma_t}(1+w)$$

$$\frac{1}{G_s} + \frac{w}{S} = \frac{\gamma_w}{\gamma_t}(1+w)$$

or $$S = \frac{w}{\frac{\gamma_w}{\gamma_t}(1+w) - \frac{1}{G_s}}$$

Summary

1. $W_s = \frac{W}{1+w}$
2. $n = \frac{e}{1+e}$
3. $Se = wG$
4. $\gamma_t = \left(\frac{G+Se}{1+e}\right)\gamma_w$
5. $\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$
6. $\gamma_d = \frac{G\gamma_w}{1+e}$
7. $\gamma' = \left(\frac{G-1}{1+e}\right)\gamma_w$
8. $\gamma_d = \frac{(1-n_a)G\gamma_w}{1+wG}$
9. $\gamma_d = \frac{\gamma_t}{1+w}$
10. $S = \frac{w}{\frac{\gamma_w}{\gamma_t}(1+w) - \frac{1}{G}}$

Example 2.1 A soil sampler of volume 1000 cm^3 is used to collect soil samples. It was found that sampler contain 2 kg soil with dry unit weight of 1800 kg/m^3. If 300 g water is mixed to the soil, then what will be the water content of the sample?

Solution:

$$\gamma_d = 1800 \text{ kg/m}^3$$

Dry weight of soil (weight of soil solids),

$$W_s = V \times \gamma_d$$

$$= 1000 \times 10^{-6} \times 1800 = 1.8 \text{ kg}$$

But actual weight of sample, $W = 2$ kg

∴ Weight of water before mixing additional water,

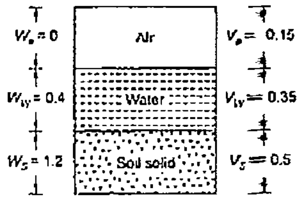
$$W_{w_1} = W - W_s = 2 - 1.8 = 0.2 \text{ kg}$$

After mixing 300 g of water, the total weight of water would be

$$W_{w_2} = 0.2 + 0.3 = 0.5 \text{ kg}$$

Thus, water content, $$w = \frac{W_{w_2}}{W_s} \times 100 = \frac{0.5}{1.8} \times 1000 = 27.8\%$$

Example 2.2 Figure shows a block diagram for a soil sample, having volumes and weights in *cc* and *g* respectively.



Consider the following statements

1. Soil is partially saturated with degree of saturation greater than 60%
2. Void ratio = 100%
3. Water content = 40%
4. Saturated unit weight = 2g/cc

Which of the above statements are TRUE?

(a) 1, 2 and 3
(b) 1, 2, 3 and 4
(c) 1, 2 and 4
(d) 2 and 3

Ans.(c)

1. Degree of saturation, $S = \frac{V_w}{V_v} \times 100 = \frac{0.35}{(0.15+0.35)} \times 100 = 70\%$

2. Void ratio, $e = \frac{V_v}{V_s} = \frac{0.15+0.35}{0.5} \times 100 = 100\%$

3. Water content, $w = \frac{W_w}{W_s} \times 100 = \frac{0.4}{1.2} \times 100 = 33.33\%$

4. Saturated unit weight, $\gamma_{sat} = \left(\frac{G+Se}{1+e}\right)\gamma_w$

Using, $Se = wG$

$$G = \frac{Se}{w} = \frac{1\times1}{0.333} = 3.0$$

$$\therefore \quad \gamma_{sat} = \left(\frac{3+1\times1}{1+1}\right)\times1 = 2 \text{ g/cc}$$

Hence option (c) is correct.

Example 2.3 An oven dry soil has mass specific gravity of 1.5 g/cc. If bulk density of soil in its natural state is 2.0 g/cc, then the water content of soil in natural state will be

(a) 50% (b) 25%
(c) 100% (d) 33.33%

Ans.(d)

For oven dry soil,

Mass specific gravity, $G_m = \frac{\gamma_d}{\gamma_w} = 1.5$ g/cc

$\therefore$ $\gamma_d = 1.5$ g/cc

Given, bulk density, $\gamma_t = 2.0$ g/cc

Using, $\gamma_d = \frac{\gamma_t}{1+w}$

$$1.5 = \frac{2.0}{1+w}$$

$$w = \frac{2.0}{1.5} - 1 = 33.33\%$$

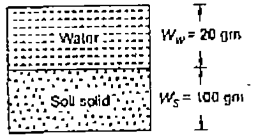
Hence option (d) is correct.

Example 2.4 Consider the phase diagram of the soil given below:

The soil is completely saturated

The specific gravity of soil solids is 2.6 (take unit weight of water as 10 kN/m³).

Match List-I (Physical properties of soil) with List II (Corresponding values) and select the correct answer using the codes given below.



List-I	List-II
1. Water content	A. 0.34
2. Void ratio	B. 0.52
3. Porosity	C. 20.53
4. Saturated density	D. 20%

Codes:

	1	2	3	4
(a)	A	C	B	D
(b)	B	A	D	C
(c)	C	A	C	D
(d)	D	B	A	C

Ans.(d)

1. Water content, $w = \frac{W_w}{W_s} \times 100 = \frac{20}{100} \times 100 = 20\%$

2. Void ratio, $e = \frac{wG}{S} = \frac{0.2\times2.6}{1} = 0.52$

3. Porosity, $n = \frac{e}{1+e} = \frac{0.52}{1+0.52} = 0.34$

4. We know, $\gamma_t = \left(\frac{G+Se}{1+e}\right)\gamma_w$

For saturated density, $\gamma_{sat} = \left(\frac{G+1\times e}{1+e}\right)\gamma_w = \left(\frac{2.6+0.52}{1+0.52}\right)\times 10 = 20.53 \text{ kN/m}^3$

Hence option (d) is correct.

Example 2.5 Soil sample *A* and *B* have void ratio of 0.5 and 0.7 respectively. If 1.5 m³ of soil sample *A* and 1.7m³ of sample *B* are mixed to form sample *C* having a volume of 3.2 m³, which one of the following correctly represents the porosity of sample *C*?

(a) 50% (b) 37.5%
(c) 100% (d) 33.33%

Ans.(b)

Soil *A*(1)	Soil *B*(2)	Soil *C*(3)
$e_1 = 0.5$	$e_2 = 0.7$	$V_3 = 3.2 \text{ m}^3$
$V_1 = 1.5 \text{ m}^3$	$V_2 = 1.7 \text{ m}^3$	

Even after mixing, volume of solids of each soil will be same in mixed sample

The solids in soil *A*, $V_{s_1} = \frac{V_1}{1+e_1} = \frac{1.5}{1+0.5} = 1 \text{m}^3$

The solids in soil *B*, $V_{s_2} = \frac{V_2}{1+e_2} = \frac{1.7}{1+0.7} = 1 \text{m}^3$

Total volume of solids in soil *C*, $V_{s_3} = V_{s_1} + V_{s_2} = 2 \text{ m}^3$

$\therefore$ Volume of void in soil *C*, $V_v = V_3 - V_{s_3} = 3.2 - 2 = 1.2 \text{ m}^3$

$\therefore$ Porosity, $n = \frac{V_v}{V_3} = \frac{1.2}{3.2} = 0.375$

Hence, option (b) is correct.

Example 2.6 Which one of the following is the water content of the mixed soil made from 1 kg of soil (say *A*) with water content of 100% and 1 kg of soil (say *B*) with water content of 50%?

(a) 66% (b) 71%

(c) 75% (d) 82%

Ans. (b)

Soil $A(1)$	Soil $B(2)$	Soil $C(3)$
$W_1 = 1$ kg	$W_2 = 1$ kg	$W_3 = W_1 + W_2$
$w_1 = 100\%$	$w_2 = 50\%$	$w_3 = ?$

Water content,
$$w = \frac{W_w}{W_s} \times 100 \Rightarrow \frac{w}{100} = \frac{W_w}{W_s}$$
$$\frac{w}{100} + 1 = \frac{W_w + W_s}{W_s} \Rightarrow W = \left(\frac{w\%}{100} + 1\right)W_s = (w+1)W_s$$

Weight of solids in soil *A*,
$$W_{S_A} = \frac{1000}{1+1} = 500 \text{ g}$$
$$W_{w_A} = 1000 - 500 = 500 \text{ g}$$

Weight of solids in soil *B*,
$$W_{S_B} = \frac{1000}{(0.5+1)} = 666.7 \text{ g}$$
$$W_{w_B} = 1000 - 666.7 = 333.3 \text{ g}$$
$$W_{S,mix} = W_{S_A} + W_{S_B} = 500 + 666.7 \text{ and } = 1166.7 \text{ g}$$
$$W_{w,mix} = W_{w_A} + W_{w_B} = 500 + 333.3 \text{ g} = 833.3 \text{ g}$$
$$\therefore \quad w = \frac{W_{w,mix}}{W_{s,mix}} = \frac{833.3}{1166.7} \times 100 = 71\%$$

Example 2.7 A soil sample has void ratio of 35%. The specific gravity of solids is 2.7. Calculate the

(i) Porosity (ii) Dry density

(iii) Unit weight if the soil is 75% saturated (iv) Unit weight if the soil is submerged

Solution:

Given, Void ratio, $e = 35\%, \quad G_s = 2.7$

(i) We know, Porosity, $n = \frac{e}{1+e}$

$$\therefore \quad n = \frac{0.35}{1+0.35} = 0.259$$

(ii) We know, $\gamma_t = \left(\frac{G_s + Se}{1+e}\right)\gamma_w$

For dry density, $S = 0$

$$\therefore \quad \gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.35}$$
$$\Rightarrow \quad \gamma_d = 19.62 \text{ kN/m}^3$$

(iii) When soil is 75% saturated i.e. $S = 0.75$

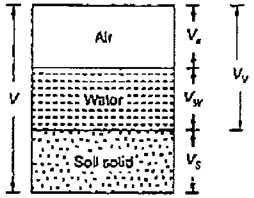
$$\therefore \quad \gamma = \left(\frac{G_s + Se}{1+e}\right)\gamma_w = \left(\frac{2.7 + 0.75 \times 0.35}{1+0.35}\right) \times 9.81 = 21.53 \text{ kN/m}^3$$

(iv) We know, $\gamma_{sub} = \gamma_{sat} - \gamma_w$

$$= \left(\frac{G_s + 1e}{1+e}\right)\gamma_w - \gamma_w = \frac{(G_s - 1)}{1+e}\gamma_w = \frac{(2.7-1)}{1+0.35} \times 9.81 = 12.35 \text{ kN/m}^3$$

Example 2.8 A sampler with a volume of 60 cm³ is filled with saturated soil sample. The specific gravity of soil solids is 2.65. When the oven dry soil is poured into a graduated cylinder filled with water, it displaces 40 cm³ of water. What is the natural moisture content and dry unit weight of soil?

Solution:



Volume of soil sample = Volume of sampler

$$\therefore \quad V = 60 \text{ cm}^3$$

When soil sample is poured into graduated cylinder, it displaces 40 cm³ of water

$$\therefore \text{ Volume of soil solids,} \quad V_s = 40 \text{ cm}^3$$
$$\because \text{ Total volume,} \quad V = V_v + V_s$$
$$\Rightarrow \quad V_v = V - V_s = 60 - 40$$
$$\Rightarrow \quad V_v = 20 \text{ cm}^3$$
$$\therefore \text{ Void ratio,} \quad e = \frac{V_v}{V_s} = \frac{20}{40} = 0.5$$

Now, we have, $e = 0.5, \quad S = 1 \quad$ and $\quad G_s = 2.65$

Using $Se = wG_s$

Moisture content, $w = \frac{Se}{G_s} = \frac{1 \times 0.5}{2.65} = 18.86\%$

We know, $\gamma_d = \frac{\gamma_t}{1+w}$

where, γ_t = bulk unit weight

$$\gamma_t = \left(\frac{G + Se}{1+e}\right)\gamma_w = \left(\frac{2.65 + 1 \times 0.5}{1+0.5}\right) \times 1 = 2.1 \text{ g/cc}$$

Hence, $\gamma_d = \frac{2.1}{1+0.1886} = 1.76$ g/cc

Example 2.9 The void ratio and specific gravity of a sample of clay are 0.73 and 2.7 respectively. The voids are 92% saturated. Find the bulk density, dry density and the water content. What would be the water content for complete saturation, the void ratio remaining the same?

Solution:

Given, $e = 0.73, \quad G = 2.7 \quad$ and $\quad S = 92\%$

We know that,

Bulk density, $\rho_t = \left(\frac{G + Se}{1+e}\right)\rho_w = \left(\frac{2.7 + 0.92 \times 0.73}{1+0.73}\right) \times 1 \quad [\because \rho_w = 1 \text{ g/cc}]$

$$\rho_t = 1.948 \text{ g/cc}$$

We know, Dry density, $\rho_d = \frac{\rho_t}{1+w}$ ...(i)

Using $Se = wG$

$\Rightarrow$ $w = \frac{Se}{G} = \frac{0.92 \times 0.73}{2.7} = 0.2487$ or 24.87%

Substituting the value of w in (i), we get,

$$\rho_d = \frac{1.948}{1+0.2487} = 1.560 \text{ g/cc}$$

Let, water content is w for full saturation at given void ratio.

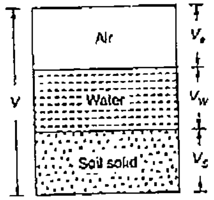
Now, $S = 100\%$, $e = 0.73$

We know, $Se = wG$

$\Rightarrow$ $w = \frac{1 \times 0.73}{2.7} = 0.2703$ or 27.03%

Example 2.10 : A compacted cylindrical specimen of 50 mm diameter and 100 mm long is to be prepared from dry soil. If the specimen is required to have a water content of 15% and the percentage of air void is 20%, calculate the weight of soil and water required in the preparation of the soil whose specific gravity is 2.69.

Solution:



Volume of soil sample = Volume of cylinder

$\therefore$ $V = \frac{\pi}{4}D^2H = \frac{\pi}{4} \times (5)^2 \times 10 = 196.35$ cc

Water content, $w = \frac{W_w}{W_s} = \frac{V_w \times \gamma_w}{V_s \times \gamma_s} = \frac{V_w}{V_s G}$

$$0.15 = \frac{V_w}{V_s G}$$

$\therefore$ $V_w = 0.15\, V_s \times G = 0.15 \times V_s \times 2.69 = 0.40\, V_s$

Also,

% Air voids, $n_a = \frac{V_a}{V} = 0.2$

$\therefore$ $V_a = 0.2\,V$

$\because$ $V = V_s + V_w + V_a$

$\Rightarrow$ $V = V_s + 0.4\,V_s + 0.2\,V$

$0.8\,V = 1.4\,V_s$

$\therefore$ $V_s = \frac{0.8\,V}{1.4} = \frac{0.8 \times 196.35}{1.4} = 112.20$ cc

Let W_s be the weight of soil solids to prepare soil whose specific gravity is 2.69.

We know, $G = \frac{W_s}{V_s \times \gamma_w}$

$\therefore$ $W_s = G \times V_s \times \gamma_w = 2.69 \times 112.2 \times 1 = 301.82$ g

Weight of dry soil = Weight of solids

= 301.82 g

Let W_w be the weight of water to prepare soil whose specific gravity is 2.69.

$$W_w = w \times W_s = 0.15 \times 301.82 = 45.27 \text{ g}$$

Example 2.11 A soil sample of saturated clay has a diameter of 50 mm and the height of 100 mm. The mass of saturated sample is 220 g and its mass when oven dried is 150 g. Find

(i) Water content of the clay
(ii) Void ratio
(iii) Dry density of solid
Assume specific gravity of solid as 2.7.

Solution:

Mass of saturated soil sample, $M_{sat} = 220$ g

Mass of oven dry soil sample, $M_s = 150$ g

$\therefore$ Mass of water in soil, $M_w = 220 - 150 = 70$ g

(i) Water content of sample, $w = \frac{W_w}{W_s} \times 100 = \frac{M_w}{M_s} \times 100$

$$= \frac{70}{150} \times 100 = 46.67\%$$

(ii) Volume of sample, $V = \frac{\pi}{4}D^2L = \frac{\pi}{4}(5)^2 \times 10 = 196.35$ cc

and $V_w = \frac{M_w}{\rho_w} = \frac{70 \text{ g}}{1 \text{ g/cc}} = 70$ cc

For a saturated soil sample,

$V_v = V_w = 70$ cc

$\therefore$ $V_s = V - V_w$

$= 196.35 - 70$

$= 126.35$ cc

$\therefore$ Void radio, $e = \frac{V_v}{V_s} = \frac{70}{126.35} = 0.55$

(iii) Dry density, $\rho_d = \frac{G\rho_w}{1+e}$

$= \frac{2.7 \times 1}{1+0.55}$

$= 1.74$ g/cc

Example 2.12 A clayey soil with specific gravity of 2.70 has natural moisture content of 16% at 70% degree of saturation. What will be its water content if, after soaking, degree of saturation becomes 90%.

Solution:

In the first case,

Given, $G = 2.7$, $w = 16\%$ and $S = 70\%$

We know, $e = \frac{wG}{S} = \frac{0.16 \times 2.7}{0.7} = 0.617$

In the second case,

Since degree of saturation is within 100%, void ratio (*e*) will be same as the previous case

$$Se = wG$$

$$w = \frac{Se}{G} = \frac{0.9 \times 0.617}{2.7} = 0.2057 \text{ or } 20.57\%$$

Example 2.13 A soil sample has wet density of 20 kN/m³ and dry density of 18 kN/m³. If the specific gravity of soil is 2.67. Calculate the void ratio, porosity, moisture content and degree of saturation. Assume unit weight of water = 10 kN/m³.

Solution:

Given, $\gamma_{wet} = \gamma_t = 20 \text{ kN/m}^3$, $\gamma_d = 18 \text{ kN/m}^3$ and $G = 2.67$

We know, $\gamma_d = \frac{G\gamma_w}{1+e}$

$\therefore$ $18 = \frac{2.67 \times 10}{1+e}$

$\Rightarrow$ $1 + e = \frac{2.67 \times 10}{18} = 1.483$

$\therefore$ $e = 0.483$

We know that,

Porosity, $n = \frac{e}{1+e} = \frac{0.483}{1+0.483} = 0.326$

We have,

Dry density, $\gamma_d = \frac{\gamma_t}{1+w}$

$\therefore$ $18 = \frac{20}{1+w}$

$$1 + w = \frac{20}{18} = 1.111$$

$$w = 0.111 \text{ or } 11.11\%$$

Using, $Se = wG$

$$S = \frac{wG}{e} = \frac{0.111 \times 2.67}{0.483} = 0.614 \text{ or } 61.4\%$$

8. Problems of Borrow Pit and Fill

Example 2.14 An embankment is to be constructed. The soil is to be compacted at maximum dry density of 18 kN/m³ at optimum moisture content = 15%. The in-situ bulk density and water content in borrow pit are 17 kN/m³ and 8% respectively. How much excavation should be carried out in the borrow pit for each cubic meter of embankment? Assume $G = 2.7$ and $\gamma_w = 10$ kN/m³.

Solution:

Borrow Pit (1)	Embankment (2)
$\gamma_1 = 17 \text{ kN/m}^3$	$\gamma_{d_2} = 18 \text{ kN/m}^3$
$w_1 = 0.08$	$w_2 = 0.15$
$G = 2.7$	$\gamma_w = 10 \text{ kN/m}^3$

For embankment $\gamma_{d_2} = \frac{\gamma_2}{1+w_2} \Rightarrow \gamma_2 = \gamma_{d_2} \times (1+w_2) = 18 \times (1+0.15)$

$\Rightarrow$ $\gamma_2 = 20.7 \text{ kN/m}^3$

Void Ratios:

Borrow Pit (1)	Embankment (2)
$\gamma_1 = \frac{G(1+w_1)}{1+e_1}\gamma_w$,	$\gamma_2 = \frac{G(1+w_2)}{1+e_2}\gamma_w$
$\therefore$ $17 = \frac{2.7(1+0.08) \times 10}{1+e_1}$,	$20.7 = \frac{2.7(1+0.15) \times 10}{1+e_2}$
$e_1 = 0.7153$ and	$e_2 = 0.5$

Volume of solids (V_s) will remain same in both borrow pit and embankment.

$\therefore$ $V_s = \frac{V_1}{1+e_1} = \frac{V_2}{1+e_2}$

$\Rightarrow$ $V_1 = \left(\frac{1+e_1}{1+e_2}\right)V_2$

$\Rightarrow$ $V_1 = \left(\frac{1+0.7153}{1+0.5}\right) \times 1 \text{ cum}$

$\Rightarrow$ $V_1 = 1.143 \text{ cum}$

Hence, the volume of excavation from borrow pit is 1.143 m³ for each cubic meter of embankment.

Example 2.15 The soil in a borrow pit has a void ratio of 0.90. A fill of volume 20000 m³ has to be constructed with an in place dry density of 19.2 kN/m³. If the owner of borrow pit to be compensated at rate 2.5 ₹ per cubic metre of excavation, determine the cost of compensation.

Take $G = 2.68$, $\gamma_w = 9.81$ kN/m³.

Solution:

Borrow Pit (1)	The fill (2)
$e_1 = 0.90$	$\gamma_2 = 18 \text{ kN/m}^3$
$G = 2.68$	$\gamma_{d_2} = 19.2 \text{ kN/m}^3$
$\gamma_w = 9.81$	

For fill, $\gamma_{d_2} = \frac{G\gamma_w}{1+e_2}$

$\therefore$ $1 + e_2 = \frac{G\gamma_w}{\gamma_{d2}} \Rightarrow \frac{2.68 \times 9.81}{19.2} = 1.37$

$\therefore$ $e_2 = 0.37$

Volume of soil solid (V_s) will remain same in both borrow pit and fill.

$$\therefore \quad V_s = \frac{V_1}{1+e_1} = \frac{V_2}{1+e_2}$$

$$\therefore \quad V_1 = \left(\frac{1+e_1}{1+e_2}\right)V_2$$

$$\Rightarrow \quad V_1 = \left(\frac{1+0.90}{1+0.37}\right)\times 20000$$

$$\Rightarrow \quad V_1 = 27737.3\,m^3$$

Cost of compensation to be given to the owner of borrow pit,

$$= 2.50\,₹/m^3 \times 27737.3\,m^3$$

$$= ₹\,69343.25$$

Example 2.16 Soil is to be excavated from a borrow pit which has a density of 1.75 gm/cc and water content of 12%. The specific gravity of soil particles is 2.7. The soil is compacted so that water content is 18% and dry density is 1.65 gm/cc. For 1000 m³ of soil in fill, estimate

(i) quantity of soil to be excavated from the pit in m³,

(ii) amount of water to be added.

Also, determine the void ratios of the soil in borrow pit and fill.

Solution:

Borrow Pit (1)	The fill (2)
$\rho_1 = 1.75$ g/cc	$V_2 = 1000\,m^3$
$w_1 = 12\%$	$w_2 = 18\%$
$G = 2.7$	$\rho_{d_2} = 1.65$ g/cc

(i)
$$\rho_{d_1} = \frac{\rho_1}{1+w_1} = \frac{G\rho_w}{1+e_1}$$

$$\Rightarrow \quad \frac{1.75}{1+0.12} = \frac{2.7\times 1}{1+e_1} \Rightarrow e_1 = 0.728$$

and
$$\rho_{d_2} = \frac{G\rho_w}{1+e_2}$$

$$1.65 = \frac{2.7\times 1}{1+e_2} \Rightarrow e_2 = 0.636$$

We know that volume of solids in borrow pit and fill is same

$$\Rightarrow \quad V_s = \frac{V_1}{1+e_1} = \frac{V_2}{1+e_2}$$

$$\therefore \quad V_1 = \left(\frac{1+e_1}{1+e_2}\right)V_2$$

$$\Rightarrow \quad V_1 = \left(\frac{1+0.728}{1+0.636}\right)\times 1000$$

$$\Rightarrow \quad V_1 = 1056.23\,m^3$$

Hence, 1056.23 m³ of soil is to be excavated from the borrow pit.

(ii) We know that,

$$V_{s_2} = \frac{V_2}{1+e_2} = \frac{1000}{1+0.636} = 611.25\,m^3$$

$$\therefore \quad \text{weight of solids} = W_s = V_s\gamma_s$$

$$\Rightarrow \quad W_s = V_sG\gamma_w$$

$$\Rightarrow \quad W_s = 611.25\times 2.7\times 9.81$$

$$\therefore \quad W_s = 16190.18\,kN$$

Now, moisture content, $w_1 = \dfrac{W_{w_1}}{W_s}$

$$\Rightarrow \quad W_{w_1}\text{ (weight of water in pit)} = w_1\times W_s = 0.12\times 16190.18 = 1942.8$$

$$\text{and} \quad W_{w_2}\text{ (weight of water in fill)} = 0.18\times 16190.18 = 2914.23\,kN$$

$$\therefore \quad \text{amount of water added} = W_{w_1} - W_{w_2}$$

$$= 2914.23 - 1942.8 = 971.43\,kN \text{ or } \frac{971.43}{9.81}\,kg$$

$$= 99.02\times 10^3\,kg$$

2.5 Relative Density (I_D or D_r)

- Relative density is also called degree of density or density index.
- It is the index which quantifies the degree of packing between the loosest and densest packing of coarse grained soil.

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} \quad \text{or} \quad \frac{\dfrac{1}{\gamma_{min}} - \dfrac{1}{\gamma}}{\dfrac{1}{\gamma_{min}} - \dfrac{1}{\gamma_{max}}}\times 100$$

where, e_{max} = void ratio in loosest state

e_{min} = void ratio in densest state

e = void ratio in natural state

- It is the most important property of a coarse grained soil.
- Coarse grained soil can be classified on the basis of relative density as given below.

I_D (%)	0 - 15	15 - 35	35 - 65	65 - 85	85 - 100
Classification	Very loose	Loose	Medium	Dense	Very loose

NOTE It is better indicator of denseness of solid than void ratio and dry unit weight, as it represents the relative compactness in absolute terms.

Soils having similar shape and grain size distribution exhibit different properties if their relative density differs.

This term is not used for cohesive soils as uncertainties are involved in computation of void ratio of cohesive soils in their loosest state in the laboratory.

Example 2.17 A saturated sand deposit have natural moisture content of 30%. It was noticed that the maximum and minimum void ratios are 0.95 and 0.40 respectively. Assume specific gravity of sand solid as 2.7. The sand deposit would be classified as

(a) medium (b) dense
(c) loose (d) very dense

Ans. (c)

Given, $w = 0.30,\ S = 1$ and $G = 2.7$

$$\therefore \quad e = \frac{wG}{S} = \frac{0.30 \times 2.7}{1} = 0.81$$

Now we have, $e = 0.81,\ e_{max} = 0.95$ and $e_{min} = 0.40$

Relative density, $$I_D = \frac{0.95 - 0.81}{0.95 - 0.40} \times 100 = 25.45\%$$

Hence, sand deposit would be classified as loose.

Example 2.18 The natural void ratio of a saturated sand sample is 0.6 and its density index is 60%. If void ratio in loosest state is 0.9, then the water content (for the sand having G = 2.7) corresponding to densest state will be

(a) 15% (b) 20%
(c) 30% (d) 50%

Ans. (a)

Given, $e = 0.6,\ I_D = 60\%$ and $e_{max} = 0.9$

Water content corresponding to densest state is given by,

$$w = \frac{Se_{min}}{G} \quad \text{...(i)}$$

$$\%I_D = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

$$0.60 = \frac{0.9 - 0.6}{0.9 - e_{min}}$$

$$\therefore \quad e_{min} = 0.4$$

Hence $$w = \frac{1 \times 0.4}{2.7} \approx 15\%$$

Hence, option (a) is correct.

Example 2.19 A relative density test was conducted on a sandy soil. The following results were obtained:

$e_{max} = 0.95,\ e_{min} = 0.40,\ I_D = 45\%$ and $G = 2.7$

Calculate the dry density of soil. If a 5 m thickness of this stratum is densified to a $I_D = 70\%$, how much will the stratum reduce in thickness?

Solution:

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

$$45 = \frac{0.95 - e}{0.95 - 0.40} \times 100$$

$$0.95 - e = 0.2475$$

$$e \approx 0.70$$

We know, $$\gamma_t = \left(\frac{G + Se}{1 + e}\right)\gamma_w$$

For dry density, $S = 0$

$$\therefore \quad \gamma_d = \frac{G\gamma_w}{1 + e} = 1.588 \text{ g/cc}$$

Considering unit surface area of sample stratum,

$$\therefore \quad e = \frac{V_v}{V_s} = \frac{V_1 - V_s}{V_s}$$

$$0.70 = \frac{5 - V_s}{V_s} \qquad [\because V_1 = 5 \times 1 \times 1 = 5 \text{ m}^3]$$

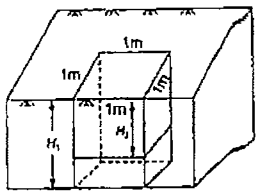
$$\therefore \quad V_s = 2.94 \text{ m}^3$$

If soil stratum is densified to 70% relative density,

$$70 = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100$$

$$0.70 = \frac{0.95 - e}{0.95 - 0.40}$$

$$e = 0.565$$



Also, $$e = \frac{V_v}{V_s} = \frac{V_2 - V_s}{2.94}$$

$$\therefore \quad 0.565 = \frac{V_2 - 2.94}{2.94}$$

$$1.661 = V_2 - 2.94$$

$$V_2 = 4.601 \text{ m}^3$$

Therefore, the reduction in thickness is given by

$$H_1 - H_2 = \Delta H = \left(\frac{V_1 - V_2}{A}\right) = \left(\frac{5 - 4.601}{1}\right) = 0.399 \text{ m} \simeq 0.4 \text{ m}.$$

Example 2.20 The density of a sand backfill was determined by field measurements to be 1746 kg/m³. The water content at the time of test was 8.6 percent and the unit weight of the solid constituents was 2.6 g/cm³. In the laboratory the void ratios in loosest and densest states was found to be 0.642 and 0.462 respectively. What was the relative density of the fill?

Solution:

Given, $\gamma = 1746 \text{ kg/m}^3,\ w = 0.086,\ \gamma_s = 2.6$ g/cc

$e_{max} = 0.642$ and $e_{min} = 0.462$

Specific gravity of soil solid,

$$G = \frac{\gamma_s}{\gamma_w} = \frac{2.6}{1} = 2.6$$

Void ratio of soil in natural state may be obtained by using,

$$\gamma = \frac{G(1+w)}{1+e}\gamma_w$$

$$1746 = \frac{2.6(1+0.086)}{1+e} \times 1000$$

$$1+e = \frac{2.6(1+0.086)}{1746} \times 1000$$

$$e = 1.617 - 1 = 0.617$$

Now,

Relative density, $$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{0.642 - 0.617}{0.642 - 0.462} \times 100 = 13.89\%$$

Example 2.21 Compute the void ratio of uniformly graded coarse grained soil in

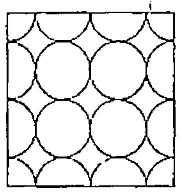
(i) loosest possible state

(ii) densest possible state

Assuming the soil particles made up of equal diameter spherical shape.

Solution:

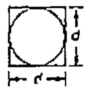
(i) In loosest packing, each sphere makes contact with six adjacent sphere-four from side, one from bottom and one from top. Assume each sphere is fitted within a cube of side d as shown in figure.



(a) Simple packing of spheres

Volume of void, V_v = Volume of cube – volume of sphere

$$= d^3 - \frac{\pi d^3}{6}$$

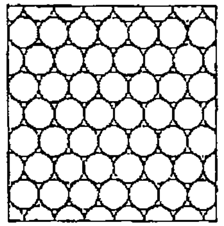


(b) Cubic element from simple packing

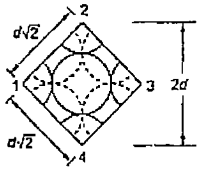
Thus, the void ratio, $$e = \frac{V_v}{V_s} = \frac{\left[d^3 - \frac{\pi d^3}{6}\right]}{\frac{\pi d^3}{6}} = \frac{\left(1 - \frac{\pi}{6}\right)}{\frac{\pi}{6}} = 0.91$$

Thus, the void ratio in loosest state of packing is 0.91.

(ii) For densest packing, particles are packed as shown in figure.



(a) Face centered cubic packing of spheres



(b) Cubic element from face centered cubic packing

Volume of void, V_v = Volume of cube – volume of sphere within cube

$$V_v = (\sqrt{2}d)^3 - \left[4 \times \frac{\pi d^3}{6}\right] = 2\sqrt{2}d^3 - \frac{2}{3}\pi d^3$$

The void ratio, $$e = \frac{V_v}{V_s} = \frac{\left[2\sqrt{2}d^3 - \frac{2}{3}\pi d^3\right]}{\frac{2}{3}\pi d^3} = \frac{2\sqrt{2} - \frac{2}{3}\pi}{\frac{2}{3}\pi} = 0.35$$

Thus, the void ratio in densest state is 0.35.

2.6 Methods for Determination of Water Content

1. Oven Drying Method

- This is the simplest and most accurate method.
- For inorganic soils, temperature is controlled between 105-110°C.
- For soil containing organic compounds, temperature is maintained about 60°C and if Gypsum is present, then temperature should be maintained at 80°C.
- Usually 4 - 6 hrs are enough for sands to dry but 16 - 20 hrs are required for clay. Usually 24 hrs are provided for drying in the oven.
- If temperature is uncontrolled and more than 110°C, there is a danger of loss of structural water.
- Water content is calculated as follows

Let W_1 = weight of empty container

W_2 = weight of container + moist soil

W_3 = weight of container + dry soil

$$W_w(\text{weight of water}) = W_2 - W_3$$

$$W_s = W_3 - W_1$$

$$\therefore \quad w = \frac{W_w}{W_s} \times 100$$

$$\Rightarrow \quad w = \frac{W_2 - W_3}{W_3 - W_1} \times 100$$

This method is accurate but time taking.

2. Pycnometer Method

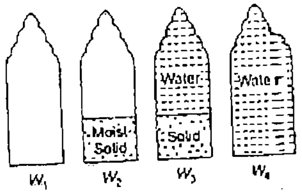


Fig. 2.5 Pycnometer method

- This is a quick method but it is less accurate than oven drying method.
- This method is used only when specific gravity of soil solids is known.
- A small weight, say 200 g to 400 g of soil is placed in a clean pycnometer whose capacity is 900 ml.
- Let W_1 = weight of empty pycnometer bottle

  W_2 = weight of pycnometer + soil

  W_3 = weight of pycnometer + soil + water

  W_4 = weight of pycnometer + water

Let G be specific gravity of soil solids,

Now, water content, $w = \frac{W_w}{W_s} \times 100$

$$\text{weight of water} = (W_2 - W_1) - W_s \quad \text{...(i)}$$

If from W_3, the weight of solids W_s could be removed and replaced by the weight of an equivalent volume of water, the weight W_4 will be

$$W_4 = W_3 - W_s + \frac{W_s}{G\gamma_w} \cdot \gamma_w$$

$$W_s = (W_3 - W_4)\frac{G}{G-1} \quad \text{...(ii)} \left[\because V_s = \frac{W_s}{\gamma_s} \text{ and } G = \frac{\gamma_s}{\gamma_w}\right]$$

from (*i*) and (*ii*),

$$w = \left[\frac{(W_2 - W_1)}{(W_3 - W_4)}\left(\frac{G-1}{G}\right) - 1\right] \times 100$$

NOTE

- In view of the difficulty in removing entrapped air from the soil sample; this method is more suited for cohesionless soil where this can be achieved easily.
- Pycnometer method is suitable for coarse grained soil. But if it is used for fine grained soil, then instead of water kerosene should be used because kerosene has good wetting properties.

3. Calcium Carbide Method/Rapid Moisture Meter Method

- It is very quick method, takes only 5 to 7 minutes but may not give accurate results.
- A soil sample weight 4 - 6 gms is placed in moisture testing equipment. The equipment consists of a closed chamber in which calibrated scale is connected to measure pressure exerted which is directly co-related to water content.
- Calcium carbide powder (CaC_2) is added on the moist soil sample which reacts with the water and as a result acetylene gas is removed which exerts pressure.

  $CaC_2 + 2H_2O \rightarrow Ca(OH)_2 + C_2H_2 \uparrow$
- The water content recorded is expressed as a % of moist weight of soil, whereas actual water content is expressed as fraction of dry weight of soil.

  Let w_r = moisture content recorded, expressed as fraction of moist weight of soil

  w = actual water content

  Then $w = \frac{w_r}{1 + w_r} \times 100\%$

4. Sand Bath Method

- It is a quick *field method.*
- This method is used when electric oven is not available.
- Soil sample is put in a container and dried by placing it on the sand bath, then it is heated over a kerosene stove.
- Water content is determined similar to oven drying method.

$$w = \frac{W_2 - W_3}{W_3 - W_1} \times 100$$

where,
W_1 = weight of empty container
W_2 = weight of container + moist soil
W_3 = weight of container + dry soil

- Since temperature is uncontrolled, hence there is a change of loss of structural water.

5. Alcohol Method

- It is also a quick method adopted in field.
- In this method, methylated sprit is mixed with the soil sample in order to increase the rate of evaporation and then methylated sprit is ignited.

$$w = \frac{W_1 - W_2}{W_2} \times 100$$

where,
W_1 = weight of sample
W_2 = weight of soil after cooling of soil + methylated sprit mixture

This method is very rapid but less accurate.

Note: Since alcohol is an oxidising agent, this method should not be used for organic soils and soils containing organic compounds.

6. Torsion Balance Method

- It is a laboratory method
- In this method, *infrared radiation is used for drying soil sample.*
- In this method, drying and weighing are done simultaneously.
- This method is rapid, accurate and most suitable for soils which quickly reabsorb moisture after drying.

7. Radiation Method

- This is an in-situ method to determine water content of soil.
- In this method, radioactive isotopes are used for the determination of water content of soil.
- A radiating device containing radioactive isotopes like Cobalt 60 is placed inside a capsule and lowered in a steel casing *A* as shown in figure. Steel casing has a small opening on one side through which rays can come out. A detector is placed inside another steel casing *B*, which also has an opening facing that of *A*.

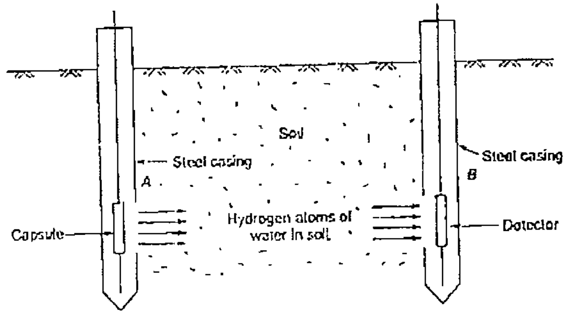


Fig. 2.6 Radiation Method

- Neutrons are emitted by the radio-active material. The hydrogen atoms in water of the soil cause scattering of neutrons. As these neutrons strike with the hydrogen atom, they lose energy. The loss of energy is proportional to the quantity of water present in the soil. The detector is calibrated to give the water contents directly.

2.7 Determination of Specific Gravity of Soil Solids

2.7.1 Pycnometer Method

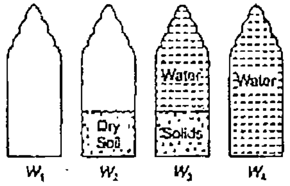


Fig. 2.7 Pycnometer method for determining specific gravity

- This method is similar to Pycnometer method for water content determination, but here oven dry soil is taken instead of moist soil.

Let W_1 = weight of empty pycnometer
W_2 = weight of pycnometer + soil sample (oven dry)
W_3 = weight of pycnometer + soil solids + water
W_4 = weight of pycnometer + water

Weight of solid, $W_s = W_2 - W_1$

Weight of equivalent volume of water

$$= (W_4 - W_1) - (W_3 - W_2)$$

$$\therefore \quad G_s = \frac{\text{weight of solid } (W_s)}{\text{weight of equivalent volume of water}}$$

$$G_s = \frac{W_2 - W_1}{(W_4 - W_1) - (W_3 - W_4)}$$

NOTE

- Specific gravity values are generally reported at 27°C (In India)
- If T°C is the test temperature, then G at 27°C is given by

$$G_{27°C} = G_{T°C} \times \frac{\text{unit weight of water at T°C}}{\text{unit weight of water at 27°C}}$$

- If kerosene (better wetting agent) is used instead of water then,

$$G = \frac{W_2 - W_1}{(W_4 - W_1) - (W_3 - W_2)} \times K$$

where, K = specific gravity of kerosene

- G can also be determined indirectly by using shrinkage limit.

Example 2.22 A Pycnometer test for the determination of water content of soil sample having specific gravity of soil solids as 2.70 yielded following data:

weight of moist soil = 800 gm
weight of pycnometer with soil and filled with water = 1875 gm
weight of pycnometer filled with water only = 1545 gm
Calculate the water content of the soil.

Solution:

Given, $W_2 - W_1$ = weight of moist soil = 800 gm
W_3 = weight of pycnometer + soil + water = 1875 gm

We know,

$$w = \left[\frac{(W_2 - W_1)}{(W_3 - W_4)} \cdot \left(\frac{G-1}{G}\right) - 1\right] \times 100$$

$$\Rightarrow \quad w = \left[\frac{\text{weight of moist sample}}{(W_3 - W_4)}\left(\frac{G-1}{G}\right) - 1\right] \times 100$$

$$\Rightarrow \quad w = \left[\frac{800}{(1875 - 1545)} \times \left(\frac{2.70 - 1}{2.70}\right) - 1\right] \times 100$$

$$\Rightarrow \quad w = [1.526 - 1] \times 100$$

$$w = 52.63\%$$

2.8 Determination of In-Situ Unit Weight

1. Core Cutter Method

- This method is used in the case of cohesive soil.
- This method cannot be used in case of hard and gravelly soils.
- Core-cutter is a cylindrical vessel with open at top and bottom with sharp edges usually having 10 cm dia and 12.5 cm height with volume nearly 1000 cc.
- The soil surface is prepared and levelled in field and cylindrical core-cutter is punched into ground and cylinder filled with soil is taken out, which is levelled at top and bottom.

Let, weight of empty core-cutter = W_1
weight of core cutter + soil = W_2
weight of soil, $W = W_2 - W_1$

Volume of soil sample, V = volume of core cutter = $\frac{\pi}{4}D^2H$

The In-situ bulk unit weight is given by,

$$\gamma_t = \frac{W}{V}$$

- If water content is known in laboratory, the dry unit weight can also be computed as

$$\gamma_d = \frac{\gamma_t}{1+w}$$

Example 2.23 A core cutter 12.6 cm in height and 10.2 cm in diameter weighs 1071 g when empty. It is used to determine the in-situ unit weight of an embankment. The weight of core cutter full of soil is 2970 g. If the water content is 6%, then

(i) What are the in-situ dry unit weight and porosity?
(ii) If the embankment gets fully saturated due to heavy rain what will be the increase in water content and bulk unit weight, if no volume change occurs?

The specific gravity of the soil solids is 2.69.

Solution:

(i) Given,

Mass of empty core cutter, $M_1 = 1071$ g

Mass of core cutter + soil sample, $M_2 = 2370$ g

∴ mass of soil sample,

$$M = M_2 - M_1$$
$$M = 2970 - 1071$$
$$M = 1899 \text{ g}$$

Volume of soil sample, V = volume of core cutter

$$= \frac{\pi}{4}D^2H = \frac{\pi}{4} \times (10.2)^2 \times 12.6 = 1029.58 \text{ cc}$$

In-situ bulk density, $\rho_t = \frac{M}{V} = \frac{1899}{1029.58} = 1.844$ g/cc

Given, w = 6% or 0.06

We know, $\rho_d = \frac{\rho_t}{1+w}$

∴ $\rho_d = \frac{1.844}{1+0.06} = 1.74$ g/cc

Hence, in-situ dry unit weight is given by,

$$\gamma_d = \rho_d \times \left(\frac{\gamma_w}{\rho_w}\right) = 1.74 \times \left(\frac{9.81}{1}\right)$$
$$\gamma_d = 17.07 \text{ kN/m}^3$$

(ii) Let void ratio of soil is e.

We know, $\gamma_d = \frac{G\gamma_w}{1+e}$

$$17.07 = \frac{2.69 \times 9.81}{1+e}$$
$$1 + e = \frac{2.69 \times 9.81}{17.04} = 1.55$$

∴ $e \simeq 0.55$

When embankment is fully saturated then $S = 1$,
Let w_{sat} is moisture content of soil at saturation,
We know, $Se = w_{sat}G$

$$w_{sat} = \frac{Se}{G} = \frac{1 \times 0.55}{2.69}$$

∴ $w_{sat} = 0.205$ or 20.5%

Increase in water content, $\Delta w = 20.5 - 6 = 14.5\%$

Now in-situ bulk unit weight $= \gamma_{sat}$

∴ $\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$

⇒ $\gamma_{sat} = \left(\frac{2.69+0.55}{1+0.55}\right) \times 9.81$

⇒ $\gamma_{sat} = 20.50 \text{ kN/m}^3$

Thus increase in in-situ bulk unit weight,

$$\Delta\gamma_t = \gamma_{sat} - \gamma_t = 20.50 - 17.07$$

∴ $\Delta\gamma_t = 3.43 \text{ kN/m}^3$

2. Sand Replacement Method

- This method is used in case of hard and gravelly soils where core-cutter method can not be used.
- In this method, a small pit or hole is made in the ground. The excavated soil is collected in the tray and weigted, say W. Now the pit is filled with sand which is measured through calibrated cylinder. Volume of soil, V = volume of sand required to fill pit completely.
- In-situ unit weight is obtained by dividing weight of excavated soil with volume of hole.

$$\gamma = \frac{W}{V}$$

- The water content of the excavated soil is also determined and the dry unit weight is obtained by using

$$\gamma_d = \frac{\gamma}{1+w}$$

- This method is widely adopted in construction of highways.

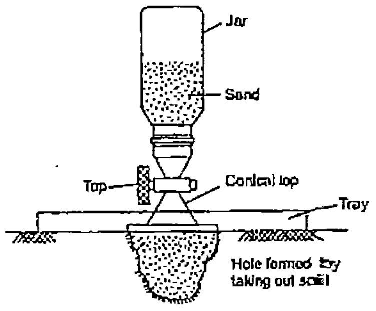


Fig. 2.8 Sand replacement method

Example 2.24 During the soil investigation for a new highway, the following observations were taken at site for the determination of unit weight by sand replacement method.

Weight of excavated soil = 452.30 g
Weight of sand + cylinder (W_1) = 10300 g
Weight of sand + cylinder after pouring the excavated hole and cone (W_2) = 9480 g
Weight of sand + cylinder after pouring for the cone only (W_3) = 9015 g
Weight of sand in calibrating can of 1000 cc after pouring from cylinder = 1580 g
Calculate the in-situ unit weight of the soil.

Solution:

Weight of sand filling the excavation hole and cone = $W_1 - W_2$ = 10300 − 9480 = 820 g
Weight of sand filling cone only = $W_2 - W_3$ = 9480 − 9015 = 465 g
Weight of sand filling hole only = 820 − 465 = 355 g

Unit weight of sand, $\gamma_{sand} = \frac{\text{weight of sand in cylinder}}{\text{volume of cylinder}}$

$$\gamma_{sand} = \frac{1580}{1000} = 1.58 \text{ g/cc}$$

Volume of excavated soil = volume of hole

$$V = \frac{\text{weight of sand filled in hole}}{\gamma_{sand}} = \frac{355}{1.58} = 224.68 \text{ cc}$$

Thus, in-situ unit weight, $\gamma_{sat} = \frac{W}{V} = \frac{452.30}{224.68} = 2.01$ g/cc

3. Water Displacement Method

- This method is suitable for highly cohesive and sticky type of soil, where it is possible to have a lump sample.
- A small soil sample is taken.
  Let weight of soil sample is = W_1
- Wax is coated over the soil sample and is immersed in a water filled cylinder which displaces water equal to volume of soil + volume of wax say V_w. Let the weight of soil sample + wax = W_2.
  Let unit weight of wax is γ_{wax}. Hence volume of wax is equal to

$$V_{wax} = \frac{W_{wax}}{\gamma_{wax}} = \frac{W_2 - W_1}{\gamma_{wax}}$$

Now, volume of soil sample.

$$V = \text{Volume of water displaced} - V_{wax}$$

$$V = V_w - \frac{(W_2 - W_1)}{\gamma_{wax}}$$

Now, bulk unit weight of soil,

$$\gamma = \frac{W_1}{V}$$

Example 2.25 **The weight of soil coated with thin layer of paraffin wax is 690.6 g and the soil alone weighs 683 g. When the soil sample coated with wax is immersed in water, it displaces 350 ml of water. The specific gravity of soil is 2.73 and that of wax is 0.89. Find the degree of saturation and void ratio if soil has water content of 17%.**

Solution:

Mass of soil alone, $M_1 = 683$ g

Mass of soil + wax, $M_2 = 690.6$ g

$$\text{Volume of soil + wax} = \text{volume of water displaced} = V_w$$
$$= 350 \text{ ml}$$

$$\therefore \quad \text{Mass of wax alone, } M_{wax} = M_2 - M_1$$
$$= 690.6 - 683 = 7.6 \text{ g}$$

$$\text{Density of wax, } \rho_{wax} = G_{wax} \times \rho_{water}$$
$$= 0.89 \times 1 = 0.89 \text{ g/cc}$$

$$\therefore \quad \text{Volume of wax, } V_{wax} = \frac{M_{wax}}{\rho_{wax}} = \frac{7.6}{0.89} = 8.53 \text{ cc}$$

$$\text{Volume of soil sample, } V = V_w - V_{wax}$$
$$= 350 - 8.53 = 341.47 \text{ cc}$$

$$\therefore \quad \text{Bulk density of soil, } \rho_t = \frac{M_1}{V} = \frac{683}{341.47} \approx 2 \text{ g/cc}$$

Since water content of soil sample w is 17%, corresponding dry density can be obtained by

$$\rho_d = \frac{\rho_t}{1+w} = \frac{2}{1+0.17} = 1.71 \text{ g/cc}$$

Also,
$$\rho_d = \frac{G\rho_w}{1+e}$$

$$\Rightarrow \quad 1.71 = \frac{2.73 \times 1}{1+e}$$

$$\therefore \quad 1 + e = \frac{2.73 \times 1}{1.71} = 1.596$$

$$e = 0.596$$

Also,
$$Se = wG$$

$$S = \frac{wG}{e} = \frac{0.17 \times 2.73}{0.596}$$

$$S = 0.778 \text{ or } 77.8\%$$

2.9 Index Properties of Soils

In nature, soil occur in different forms. However soils exhibiting similar behaviour can be put into a particular category. Different tests are done to assess the engineering behaviour of soils.

Index properties are those properties which are used for the identification and classification of soils.

Index properties can be divided into two general types:

(i) Soil grain properties

(ii) Soil aggregate properties

The soil grain properties depend on the individual grains of soil mass whereas. soil aggregate properties depends on the soil mass as a whole i.e. soil history, mode of formation or on soil structure. Hence soil 'aggregate properties' are of great engineering importance.

(i) **Soil Grain Properties**

The most important soil grain properties of soil are:

(a) Grain size distribution: by sieve and sedimentation analysis

(b) Grain shape: Bulky, flaky and needle shaped etc.

(ii) **Soil Aggregate Properties**

The various soil aggregate properties are:

(a) Unconfined compressive strength (q_u)

(b) Consistency and atterberg's limits

(c) Sensitivity

(d) Thixotropy and soil activity

(e) Relative density

Type of Soil	Index Property
Coarse Soil	Particle size, Grain shape and Relative density
Fine Soil	Atterberg's limit and consistency, unconfined compressive strength, thixotropy and activity

2.10 Particle Size Analysis

Grain Size Distribution

- Grain size analysis of coarse grained soils is carried out by sieve analysis, whereas analysis of fine grained soils is by sedimentation method i.e. either by hydrometer or pipette method.
- Generally, most of the soil contains both coarse as well as fine grain constituents. Hence a combined analysis is usually carried out.
- In combined analysis, dry soil fraction retained on sieve size 4.75 mm is called gravel fraction which is subjected to coarse sieve analysis and soil fraction passing through 4.75 mm sieve is further subjected to fine sieve analysis.
- Fraction passing through 0.075 mm sieve is analysed by hydrometer or pipette method.

I. S. Classification

S.N.	Type of soil	Particle size	Remark
1.	Boulder	> 300 mm	Not considered as soil
2.	Cobbles	60 mm – 300 mm	
3.	Gravel	4.75 mm – 80 mm	Coarse grained soil
4.	Sand	0.075 mm – 4.75 mm	
	(4.1) Coarse sand (4.2) Medium sand (4.3) Fine sand	2 mm – 4.75 mm 0.425 mm – 2 mm 0.075 mm – 0.425 mm	
5.	Silt	0.002 mm – 0.075 mm	Fine grained soil
	(6.1) Coarse (6.2) Medium (6.3) Fine	0.02 mm – 0.075 mm 0.01 mm – 0.02 mm 0.002 mm – 0.01 mm	
6.	Clay	< 0.002 mm	

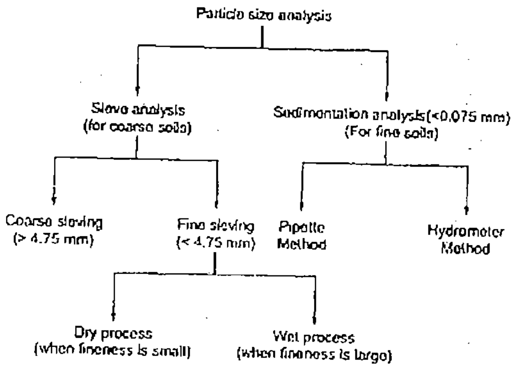


Fig.2.9

2.10.1 Sieve Analysis

1. **Coarse Sieving**

- The fraction retained on 4.75 mm sieve is called the gravel fraction and is subjected to coarse sieve analysis.
- Sieves are represented either by their number or either by size. IS sieves have square size opening represented in mm or micrometer.
- Sieve no. represents no. of square openings in 1 inch of length. For eg. 0.075 mm size sieve has Sieve no. IS-200.
- The sieves used in coarse seiving are 80mm, 20 mm, 10mm, 4.75 mm (4 No. of sieves)

- The sample is shaken for 10 min. in the shaking machine and weight of soil retained in each sieve is found.

Let,
W_i = Weight retained in the i^{th} sieve
W = Total weight of soil sample taken

$$\% \text{ weight retained on } i^{th} \text{ sieve} = \frac{W_i}{W}\times 100$$

$$\text{Cumulative \% retained} = \frac{\text{Total weight of soil retain up to } i^{th} \text{ sieve}}{\text{Total weight of soil taken}}\times 100$$

$$= \sum_{i=1}^{i}\frac{W_i}{W}\times 100$$

$$\% \text{ finer than } i^{th} \text{ sieve } (\%N) = 100 - \text{cumulative \% retained.}$$

Grain Size distribution Curve

- A graph is plotted between % finer and sieve size in semi-log paper. Sieve size (particle dia) is taken on log scale on x-axis and % finer in arithmetic scale in y-axis.
- From the grain distribution curve, size is computed corresponding to 60% finer, 30% finer and 10% finer are computed. They are represented as D_{60}, D_{30} and D_{10} respectively.
- D_{60} is that size below which 60% particles are finer than this size by weight.
- D_{30} is that size below which 30% particles are finer than this size by weight.

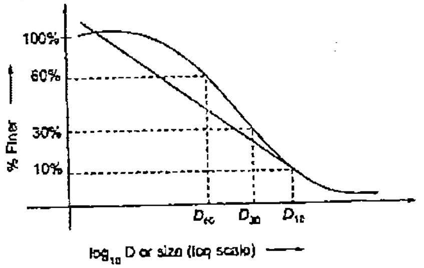


Fig. 2.10 Grain distribution curve

- D_{10} is that size below which 10% particles are finer than this size by weight. D_{10} is also called as *effective size.*
- D_{50} is called average size.
- Using D_{60}, D_{30} and D_{10}, following shape parameters are defined for the classification of coarse soils:

1. Coefficient of uniformity,

$$C_u = \frac{D_{60}}{D_{10}}$$

2. Coefficient of curvature,

$$C_C = \frac{D_{30}^2}{D_{60}\times D_{10}}$$

Typical results of a sieve analysis are shown in figure:

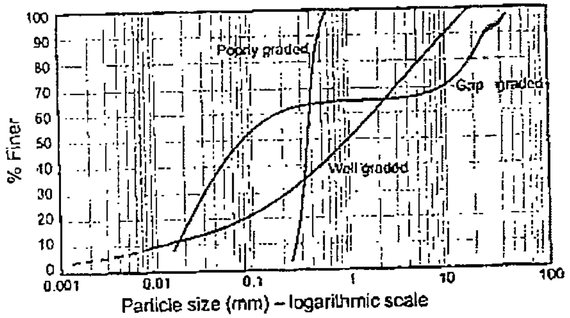


Fig.2.11 Typical Particle size distribution curve

- Well graded means soil particles of all size are present.
  - (i) For well graded gravel $C_u > 4$ and $1 \le C_C \le 3$
  - (ii) For well graded sand $C_u > 6$ and $1 \le C_C \le 3$
- Poorly graded or uniformly graded means soil particle of one size are present predominantly.
- Gap graded means soil particle of same size are missing.
- $C_u \simeq 1$ for uniformly/ poorly graded soil

2. Fine Sieving (Sand Sieving)

- Soil fraction passing through 4.75 mm sieve is subjected to fine sieve analysis.
- It can be performed either in dry state or wet state. Wet sieving is preferred when clay content is present in the sand. So sand is washed to remove the clay.
- In fine sieving, following sieves are arranged in decreasing order as 2 mm, 1 mm, 600 μ-m, 425 μ-m, 150 μ-m and 75 μm.
- The procedure of analysis is same as coarse analysis.

Example 2.26. From the results of a sieve analysis given below, plot a grain-size distribution curve and then determine.

(i) The effective size

(ii) The coefficient of uniformity

(iii) The coefficient of curvature

Weight of soil taken for sieve analysis was 500 g.

Sieve size	Weight of soil retained in each sieve, g
4.76 mm	3.8
2.40 mm	32.2
1.20 mm	52.8
0.60 mm	38.7
0.30 mm	122.5
0.15 mm	15.0
0.075 mm	26.4

Solution:

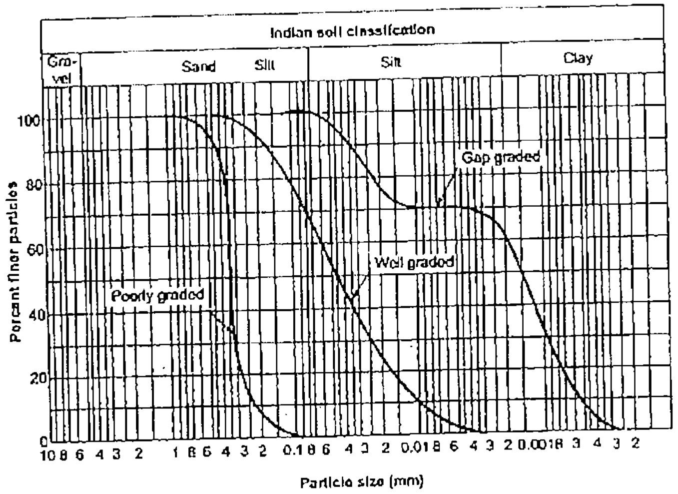
Since, total weight is 500 gm.

$$\therefore \text{ Percentage retained on each sieve} = \frac{3.8 \times 100}{500} = 0.76\%$$

Similarly for every sieve size calculation will be done as:

Sieve size (mm)	Weight retained (g)	% Retained on each sieve	% Cumulative retained	% Finer
4.76 mm	3.8	0.76	0.76	99.24
2.40 mm	32.2	6.44	7.20	92.80
1.20 mm	52.8	10.56	17.76	82.24
0.60 mm	38.7	7.74	25.50	74.50
0.30 mm	122.5	24.50	50.00	50.00
0.15 mm	159.9	31.98	81.98	18.02
0.075 mm	26.4	5.28	87.26	12.74

The grain distribution curve is plotted as shown in figure below



From the curve,

(i) Effective size, $D_{10} = 0.07$ mm

(ii) $D_{30} = 0.21$ mm and $D_{60} = 0.41$ mm

$$\therefore \text{ Coefficient of uniformity,} \quad C_u = \frac{D_{60}}{D_{10}} = \frac{0.43}{0.07} = 6.14$$

$$\text{(iii) Coefficient of curvature,} \quad C_C = \frac{D_{30}^2}{D_{60} \times D_{10}} = \frac{0.21^2}{0.43 \times 0.07} = 1.47$$

2.10.2 Sedimentation Analysis

- Sedimentation analysis is used to determine grain size distribution of the soil fraction passing through 75 μm size.
- It is based on the 'Stoke's Law'.

Stoke's Law

If a spherical particle falls through infinitely large medium, then it will achieve a constant terminal velocity. Terminal velocity is given as

$$V = \frac{(\gamma_s - \gamma_l)}{18\mu} D^2 = \frac{(\rho_s - \rho_l)}{18\eta} D^2$$

where,

γ_s = unit weight of spherical particle

γ_l = unit weight of liquid

D = dia of falling spherical particle

μ = dynamic viscosity of liquid $\frac{(N-s)}{m^2}$

η = kinematic viscosity m²/sec

Limitations of Stokes Law

1. The analysis is based on the assumption that the falling particle is spherical. But in soils, the finer particles are never truly spherical.
2. Stoke's law considers the velocity of free fall of a single sphere in a liquid of infinite extension, where as the grain size analysis is usually carried out in a glass jar in which the extent of liquid is limited.
3. The fine grains of soil carry charges on their surface and have a tendency for floc formation. If the tendency of floc formation is not prevented, the diameter measured will be the diameter of the floc and not of the individual.

NOTE

- Stokes law is applicable for spheres of diameter between 0.2 mm and 0.0002 mm.
- Spheres of diameter larger than 0.2 mm falling through water causes turbulence, whereas for spheres with diameter less than 0.0002 mm. Brownian motion takes place and the velocity of settlement is too small for accurate measurement.

Procedure of Sedimentation Analysis

First step involved is the preparation of soil sample. Soil sample is mixed with water and suspension is made.

Treatment given to soil sample

1. Pre-treatment: Treatment given before making of soil suspension to remove organic matters and calcium compounds.

   For organic matter-Oxidizing agent is used (e.g. H_2O_2)

   For calcium compounds - Acids are used (e.g. HCl)
2. Post-treatment: It is done after preparation of soil suspension to break the flocs that are formed due to presence of surface electric charges. The dispersing agents (deflocculating agent) used are sodium hexameta phosphate or calgon, sodium oxalate, etc.

   The analysis is carried out by the hydrometer or pipette method. The principle of the test is same in both methods. The difference lies only in the method of making observations.

1. **Pipette Method**
   - 10 ml sample of suspension is drawn off with a pipette from a specified depth from the surface at different time intervals
   - This 10 ml sample is put in a container and is dried in oven to get dry unit weight/dry density.

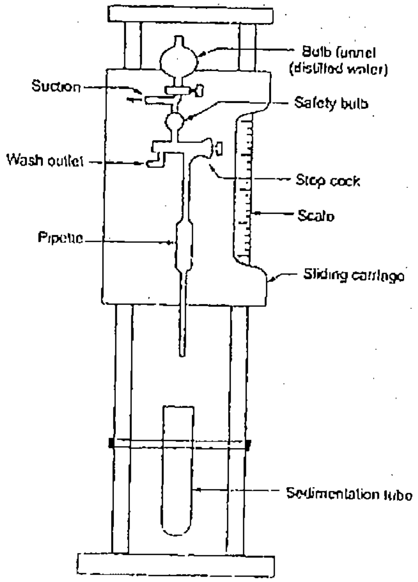


Fig. 2.12 Pipe the method

Let m_d be the mass of dried sample obtained from pipette volume (V_P = 10 ml)

Hence, mass per unit volume of dried sample

$$= \frac{m_d}{\text{Volume of piple } (V_P)} = \frac{m_d}{10ml}$$

Let M is the total mass of dry soil which is used to prepare the suspension having total volume of V.

- If dispersing agent as added in the total volume V, of mass.

  Then mass per unit weight of dispersing agent is $= \frac{m'}{V}$
- The mass per unit volume of soil solids at any time interval is given by

  $$= \frac{m_d}{V_p} - \frac{m'}{V}$$
- The percentage finer is given by

  $$\% N = \frac{\frac{m_d}{V_P} - \frac{m'}{V}}{\frac{M}{V}} \times 100$$
- The diameter of falling particle at any instance of time is given by the Stoke's law

  $$\frac{H_o}{t} = V = \frac{(\gamma_s - \gamma_l)D^2}{18\mu}$$

  where, H_o = effective depth through which particle settles.
- Observation are taken at regular intervals, a graph is plotted between % finer (%N) and diameter of particle.

Note: In pipette method, sample is collected from height H_o at various time intervals i.e. H_o is fixed.

Example 2.27 In a sedimentation analysis 73 g of soil passing 75 µm is dispersed in 1,000 ml of water. In order to estimate the percentage of particle size less than 0.002 mm, how long after the commencement of sedimentation is the pipette reading to be taken? The sample was drawn 165 mm below the surface of suspension in jar. The specific gravity of soil grain is 2.72 and viscosity of water is 0.001 N-s/m².

Solution:

Given, D = 0.002 mm, µ = 0.001 N-s/m², H_o = 165 mm

Settling velocity,
$$V = \frac{(\gamma_s - \gamma_w)D^2}{18\mu} = \frac{(G_s - 1)\gamma_w D^2}{18\mu}$$

$$= \frac{(2.72-1) \times 9.81 \times 10^3 \times (0.002 \times 10^{-3})^2}{18 \times 0.001} = 3.7496 \times 10^{-6} \text{ m/s}$$

$$\therefore \quad V = \frac{H_o}{t}$$

$$3.7496 \times 10^{-6} = \frac{165 \times 10^{-3}}{t}$$

$$t = \frac{165 \times 10^{-3}}{3.7496 \times 10{-}{-6}} = 44004.69 \text{ sec. or } 12.22 \text{ hrs.}$$

Example 2.28 In a pipette analysis, 25 g of soil was dispersed in water, and the suspension was made to a volume 500 m*l*. The viscosity of water is 0.0012 SI units. Seven minutes after the commencement of sedimentation, 10 m*l* of suspension was taken at a depth of 100 mm. The sampled volume was dried and found to have mass of 0.3 g and G = 2.70. Compute

(a) Largest size of particle remaining in suspension 7 minutes after the commencement of sedimentation at a depth 100 mm.

(b) The percentage of finer particles.

Solution:

Given, $M = 25$ g. $t = 7$ minutes or 420 sec., $V = 500$ m*l*, $V_p = 10$ m*l*

$\mu = 0.0012$, $H_o = 100$ mm, $m_p = 0.3$ g, $G = 2.70$

Using Stoke's law,
$$V = \frac{(G-1)\gamma_w D^2}{18\mu} = \frac{(G-1)g\rho_w D^2}{18\mu}$$

$$\therefore \quad \frac{H_o}{t} = \frac{(G-1)g\rho_w D^2}{18\mu}$$

$$D = \sqrt{\frac{18\mu H_o}{(G-1)g\rho_w t}} = \sqrt{\frac{18\times0.0012\times100\times10^{-3}}{(2.70-1)\times9.81\times1000\times420}}$$

$$= 1.756\times10^{-5}\text{ m or }0.0175\text{ mm}$$

$$\%\text{ finer }(\%N) = \frac{\frac{m_p}{V_p}-\frac{m'}{V}}{\frac{M}{V}}\times100 = \frac{\frac{0.3}{10}-0}{\frac{25}{500}}\times100 = 60\%$$

2. Hydrometer Method

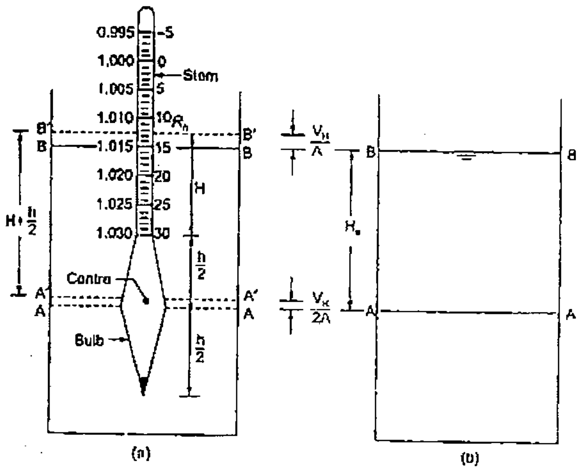


Fig. 2.13 *Hydrometer method*

- It is also based on Stoke's law.
- Hydrometer is a device used to measure specific gravity of *liquids*.
- In this method, the weight of solid present at any time is calculated directly by reading the density of soil suspension.

Calibration of hydrometer: It involves establishing a relation between the hydrometer reading R_H and effective depth (H_e).

The effective depth is the distance from the surface of the soil suspension to the level at which the density of soil suspension is being measured.

- Effective depth is calculated as

$$H_e = H_1 + \frac{1}{2}\left(h - \frac{V_H}{A_j}\right)$$

where,
H_1 = distance (cm) between any hydrometer reading and neck
h = length of hydrometer bulb
V_H = volume of hydrometer bulb
A_j = area of the cross section of the jar

- Reading of hydrometer is related to specific gravity or density of soil suspension as :

$$G_{SS} = 1 + \frac{R_H}{1000}$$

Thus a reading of R_H = 25 means, G_{SS} = 1.025
and a reading of R_H = −25 means, G_{SS} = 0.975

- % finer is given as :

$$\%\,N = \frac{G}{G-1}\rho_w\left(\frac{V}{W}\right)\left(\frac{R_H}{10}\right)\%$$

where,
$G = G_{SS}$ = Specific gravity of soil solids
R_H = Final corrected reading of Hydrometer
V = Total volume of soil suspension
W = Weight of soil mass dissolved in g

Corrections to Hydrometer Reading

Hydrometer readings observed in the test are further corrected for following:

1. Meniscus Correction (C_m)
   - Hydrometer reading is always corresponding to the upper level of meniscus.
   - Therefore, meniscus correction is always positive.
2. Temperature Correction (C_t)
   - Hydrometers are generally calibrated at 27°c. If the test temperature is above the standard (27°C) the correction is added and if below, it is subtracted.
3. Dispersing/Deflocculating agent Correction (C_d)
   - The correction due to rise in specific gravity of the suspension on account of the addition of the deflocculating agent is called dispersing agent correction (C_d)
   - C_d is always negative
   - The corrected hydrometer reading is given by

$$R_{HC} = R_H + C_m \pm C_t - C_d$$

Example 2.29 The corrected hydrometer reading in 1000 *ml* of soil suspension, 60 minute after commencement of sedimentation is 20. The effective depth from the calibrations is 165 mm. If the particle specific gravity is 2.74 and the viscosity of water is 0.01 poise, calculate

(i) The smallest particle size which would have settled during this interval of 60 minutes.

(ii) % of particle finer than this size.

The total weight of the soil sample taken was 48 grams.

Solution:

Given,

$$V = 1000\,ml$$
$$t = 60 \text{ min or } 3600 \text{ sec.}$$
$$\mu = 0.01 \text{ poise} = 0.01 \times 10^{-1} \text{ N-s/m}^2$$
$$H_e = 165 \text{ mm}$$

(i)
$$\frac{H_e}{t} = \frac{(G-1)\gamma_w D^2}{18\mu}$$

$$D = \sqrt{\frac{18\mu H_e}{(G-1)\gamma_w t}} = \sqrt{\frac{18\times0.01\times10^{-1}\times165\times10^{-3}}{(2.74-1)\times9.81\times3600}}$$
$$= 2.198 \text{ m} \times 10^{-5} \text{ or } 0.022 \text{ mm}$$

(ii) Percentage finer than particle of size *D* is given by,

$$\% N = \frac{G}{(G-1)}\rho_w \times\left(\frac{V}{W}\right)\times\left(\frac{R_H}{10}\right)\%$$

where,
G = sp.gravity of solid
V = volume of soil sample = 1000 *ml* or 1000 cc.
W = weight of dry soil used to make suspension = 48 g
R_H = corrected Hydrometer reading = 20

$$\therefore \quad \% N = \frac{2.74}{(2.74-1)}\times1\times\left(\frac{1000}{48}\right)\times\left(\frac{20}{10}\right)\% = 65.61\%$$

Example 2.30 In a hydrometer analysis. 45 gm of soil is mixed with distilled water to make 1200 *ml* suspension. After 45 sec. the reading of hydrometer is 1.015. The depth of suspension below the reading is found to be 140 mm. The dimension of hydrometer are :

Volume of hydrometer = 75 cc

The internal area of jar of Hydrometer = 60 cm²

Assuming ρ_w = 1g/cc, μ = 0.001 N-s/m² and G_s = 2.7

Find out the co-ordinate of the point on the grain size plot.

Solution:

Given,

$$V_H = 75 \text{ cc}, \quad A = 60 \text{ cm}^2$$

$$H_e = H_1 + \frac{1}{2}\left(h - \frac{V_H}{A_j}\right) = \left(H + \frac{1}{2}h\right) - \frac{V_H}{2A_j}$$

Here
$$\left(H + \frac{1}{2}h\right) = \text{Depth of suspension below the hydrometer reading}$$
$$= 140 \text{ mm or } 14 \text{ cm}$$

$$\therefore \quad H_e = 14 - \frac{75}{2\times60} = 13.375 \text{ cm}$$

By Stoke's law, we have
$$D = \sqrt{\frac{18\mu H_e}{(G-1)\gamma_w t}} = \sqrt{\frac{18\times0.001\times13.375\times10^{-2}}{(2.7-1)9.81\times45}}$$
$$D = 1.7910\times10^{-4} \text{ m or } 0.179 \text{ mm}$$

Percentage finer (% *N*) is given by,

$$\%N = \frac{G}{G-1}\rho_w\left(\frac{V}{W}\right)\left(\frac{R_H}{10}\right)\%$$

Since reading of Hydrometer is 1.015 which is density of suspension at the time of observation,

$$\therefore \quad 1.015 = 1 + \frac{R_H}{1000}$$

$$\frac{R_H}{1000} = 0.015$$

$$\therefore \quad R_H = 15$$

Hence
$$\%N = \frac{2.7}{(2.7-1)}\times1\times\left(\frac{1200}{45}\right)\times\left(\frac{15}{10}\right)\% = 63.52\%$$

Thus the coordinate of the point on the grain size curve is (0.179 mm, 63.52)

2.11 Consistency of Clays (Atterberg's Limits)

- Consistency represents relative ease with which a soil can be deformed.
- In practice, consistency is a property associated only with fine grained soils, especially clays.
- Consistency of clays is related to the water content.
- Depending on percentage water content, four stages of consistency are used to describe the state of a clayey soil :
  1. Solid State
  2. Semi Solid State
  3. Plastic State
  4. Liquid State
- The boundary between any two states is called consistency limit. They are also known as Atterberg limits after Swedish scientist Atterberg, who first demonstrated the significance of these limits.

where,
V_d = Volume of dry soil mass
= Volume of soil at shrinkage limit
V_p = Volume of soil at plastic limit
V_L = Volume of soil at liquid limit
w_L = Liquid limit
w_p = Plastic limit
w_S = Shrinkage limit

Slope,
$$\frac{dy}{dx} = \text{constant}$$

$$\Rightarrow \quad \frac{V_l - V_p}{w_L - w_p} = \frac{V_p - V_d}{w_p - w_s}$$

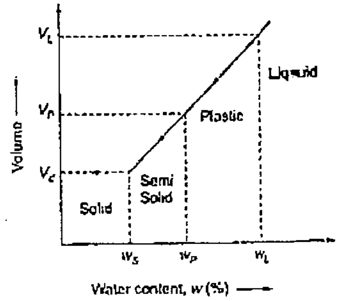


Fig. 2.14 States of clayey soil and consistency limits

- For change in water content corresponding to change in degree of saturation from 0 to 100%, there is no change in total volume of soil. But for water content increasing greater than shrinkage limit (S = 100%), then with change in water content total volume of soil also changes.
- At shrinkage limit all the pores of soil are just filled by water. Hence degree of saturation (S) is 100%.
- Naturally existing soils have water content between w_L and w_P.
- On increase in water content shear strength of soil decreases.
- At liquid limit, all soils have negligible and equal shear strength.

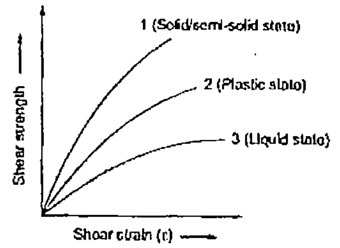


Fig. 2.15 Shear strength behaviour at different soil states

2.11.1 Liquid Limit (w_L)

It is that minimum water content at which soil has tendency to flow. At liquid limit, consistency of soil changes from plastic state to liquid state.

At liquid limit all soil have nearly negligible shear strength of 2.7 kN/m² approximately.

Determination of Liquid Limit: Liquid limit is found out by the following two methods:

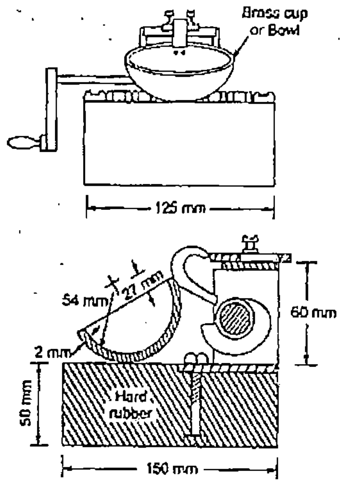
(a) Casagrande's apparatus.
(b) Cone penetration.

(a) Casagrande's apparatus

- About 120 g oven dried soil is taken and mixed with water (say w_1 %) to attain putty like consistency.
- Paste is placed inside casagrande apparatus cup and levelled.
- A groove of 2 mm size is cut and apparatus is given blows over a rubber pad and no. of blows required to close the 2 mm groove is noted as N_1.
- Now same soil is mixed with water content w_2 and no. of blows required to close the 2 mm groove is noted say N_2.
- Same process is repeated with different water content.
- A graph is plotted between % water content and No. of blows in semi log scale.
- The above curve is called flow curve and the slope of above curve is called flow index (I_f).

I_f = tan θ

$$= \frac{w_1 - w_2}{\log_{10} N_2 - \log_{10} N_1} = \frac{w_1 - w_2}{\log_{10}\left(\frac{N_2}{N_1}\right)} = \text{constant}$$



Standard liquid limit test apparatus

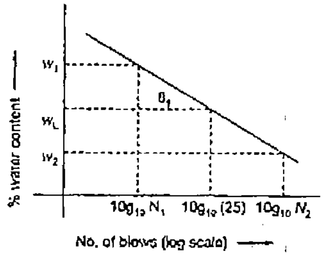


Fig. 2.16 Flow Curve

- *If a soil has greater flow index, it means that the rate of loss of shear strength with increase in water content is high.*

NOTE The Compressibility of soil directly depends on liquid limits. Greater the liquid limit, greater is the compressibility of the soil.

e.g. For soil X, w_L = 42 %
soil Y, w_L = 56 %

Soil Y is more compressible than soil X.

- Clay have more liquid limit than silt.

(b) Cone Penetrometer Method

The cone penetrometer test is another method recommended by the Indian standards to find the liquid limit.

- To find liquid limit, the penetration of standard cone into soil sample is measured for 30 sec. If the penetration is less than 20 mm, the wet soil is taken out and mixed thoroughly with water and test is repeated till the penetration is between 20 and 30 mm.
- The test is repeated for a variety of water contents, and the *water content corresponding to a penetration of 25 mm is taken as the liquid limit of the sample.*

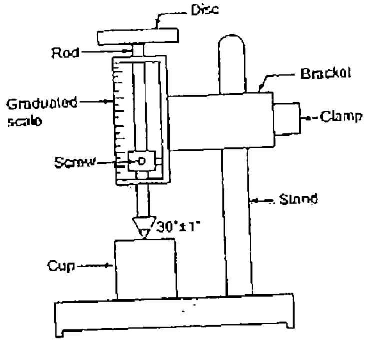


Fig. 2.17 Cone Pentrometer

Example 2.31 The relationship between water content (w%) and number of blows (N) in soils, as obtained from casagrande's liquid limit apparatus is given by

$$w = 20 - \log_{10} N$$

The liquid limit of soil is

(a) 15.6 %
(b) 16.6 %
(c) 17.6 %
(d) 18.6 %

Ans. (d)

Liquid limit is also defined as the "water content corresponding to 25 blows to close the 2 mm groove".

It means, for liquid limit of soil,

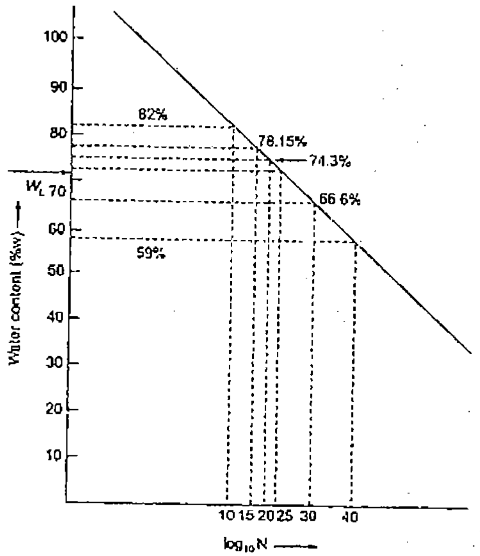
$$N = 25$$
$$w_L = 20 - \log_{10} 25 = 20 - \log_{10}(5)^2$$
$$\therefore \quad w_L = 20 - 2\log_{10} 5 = 20 - 2 \times 0.698 = 18.60\ \%$$

Example 2.32 The results were obtained from a liquid limit test of soil.

No. of blows	10	15	20	30	40
Water content	82.0%	78.15%	74.3%	66.6%	59%

(a) Plot the flow curve and determine liquid limit.
(b) Flow index of soil.

Solution:



(i) From the graph,

$$w_L = w_{20} + \frac{w_{30} - w_{20}}{\log\left(\frac{30}{20}\right)} \log\left(\frac{25}{20}\right)$$

$$w_L = 74.3 + \frac{66.6 - 74.3}{30 - 20} \times 5$$

$$w_L = 74.3 - 4.23 = 70.07\ \%$$

(ii) Flow index,

$$I_f = \frac{w_1 - w_2}{\log_{10} N_2 - \log_{10} N_1} = \frac{82 - 59}{\log_{10}\left(\frac{40}{10}\right)}$$

$$I_f = \frac{23}{2\log_{10} 2} = 38.2\%$$

2.11.2 Plastic Limit (w_p)

- The water content at which soil sample changes from semi-solid to plastic state is known as Plastic Limit.
- Plastic limit is also defined as the water content at which soil would just begin to crumble when rolled into a thread of approximately 3 mm diameter.
- Clays have high plastic limit and liquid limit.

  but $LL >> PL$

- Coarse grained soil like sand and gravel have less liquid limit and plastic limit generally.

$$w_L \simeq w_P$$

Eg.

Type of soil	Liquid limit (w_L)	Plastic limit (w_P)
Black cotton soil	400 - 500%	200 - 250%
Alluvial soil (sand)	10 - 50%	10 - 15%

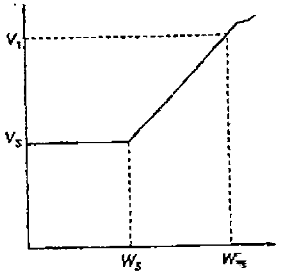
- Plastic limit depends upon amount and type of clay mineral in soil. Hence clay containing fine soils have more plastic limit.

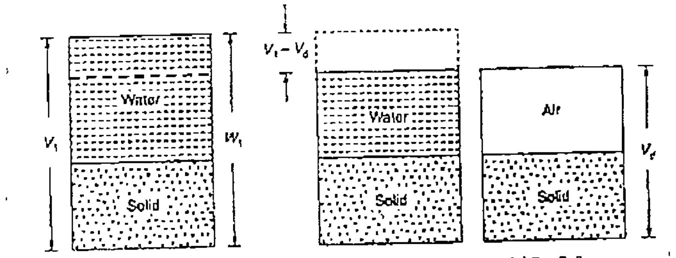
NOTE

- If organic matter is mixed with soil then LL and PL both increases but increase in LL > increase in PL
- If sand is mixed in clays then LL and PL both reduces but decrease in LL > decrease in PL

2.11.3 Shrinkage Limit (w_s)

- A state when the decrease in moisture content leads to solid state, no change in volume of soil mass is observed, the consistency of soil changes from semi-solid to solid state. The boundary water content is called shrinkage limit.
- Shrinkage limit is the smallest value of water content at which soil mass is completely saturated. It means that below shrinkage limit, soil is partially saturated.





(a) Original Soil (b) Soil at shrinkage limit (c) Dry Soil

Fig. 2.18 Determination of Shrinkage limit

where,
V_1 = original volume of soil
V_d = dry volume of soil
W_1 = original weight of soil
W_d = W_s = dry weight of soil (solids)
w_1 = natural water content

Shrinkage limit,

$$w_s = \frac{W_w}{W_s} = \frac{(W_1 - W_s) - (V_1 - V_d)\gamma_w}{W_d} = \frac{(W_1 - W_s)}{W_d} - \frac{(V_1 - V_d)\gamma_w}{W_d}$$

$$w_s = w_1 - \left(\frac{V_1 - V_d}{W_d}\right) \times \gamma_w$$

- Shrinkage limit test can be used to determine sp.gravity of soil solids

$$G = \frac{1}{\left(\frac{\gamma_w}{\gamma_d} - \frac{w_s}{100}\right)}$$

where,
γ_w = unit weight of water
γ_d = dry unit weight of soil
w_s = % shrinkage limit

- If specific gravity G and void ratio e are known, then

$$w_s = \frac{Se}{G} = \frac{e}{G} \qquad (\because S = 1)$$

- **Shrinkage ratio (R)** : It is defined as the ratio of a given volume change in a soil, expressed as a percentage of the dry volume to the corresponding change in water content above the shrinkage limit.

$$R = \frac{\left(\frac{V_1 - V_2}{V_d}\right) \times 100}{w_1 - w_2}$$

where,
V_1 = volume of soil mass at water content w_1%
V_2 = volume of soil mass at water content w_2%
V_d = volume of dry soil mass.

- Special case: At shrinkage limit.

$$w_2 = w_s \text{ and } V_2 = V_d$$

$$\therefore \quad R = \frac{\left(\frac{V_1 - V_d}{V_d}\right) \times 100}{w_1 - w_s}$$

- Shrinkage ratio can also be defined as,

$$R = \frac{\gamma_d}{\gamma_w}$$

- **Volumetric Shrinkage :** It is the percentage loss in volume of soil on drying

$$V_s = \frac{V_1 - V_d}{V_d} \times 100$$

or

$$V_s = R(w_1 - w_s)$$

- **Degree of shrinkage :** It is the percentage loss in volume of soil on drying corresponding to initial volume.

$$D.O.S. = \frac{V_i - V_d}{V_i} \times 100$$

where,
V_i = initial volume of soil sample
V_d = Dry volume of soil sample

Degree of shrinkage	< 5%	5 - 10%	10 - 15%	> 15%
Plasticity	Good	Medium	Poor	Very poor

- Linear Shrinkage : It refers to decrease in one dimension of soil expressed as % of initial dimension.

$$L_s = \left[1 - \left(\frac{100}{100 + V_s}\right)^{1/3}\right] \times 100$$

V_s = volumetric shrinkage.

Stress-strain curve for different consistency states :

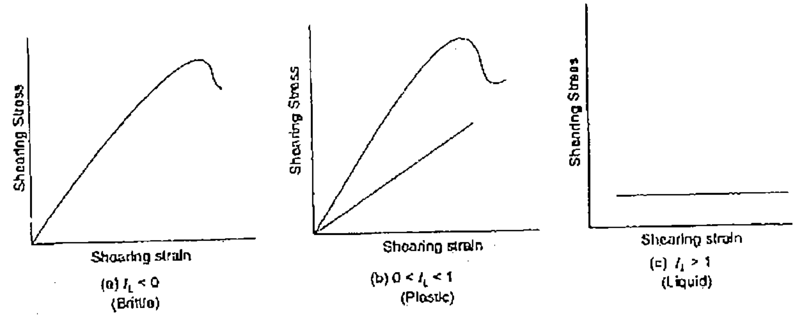


Fig.2.19

Example 2.33 An oven dry soil sample of 200 cm³ weighs 360 g. If the specific gravity is 2.7, determine the void ratio and shrinkage limit. What will be the water content which will fully saturate the sample and cause an increase in volume equal to 10% of the original dry volume.

Solution :

$V_d = 200 \text{ cm}^3$

Weight of dry soil sample, $W_d = 360$ g

Specific gravity, $G = 2.7$

Dry density of sample, $\gamma_d = \frac{W_d}{V_d} = \frac{360}{200} = 1.8$ g/cc

We know, $\gamma_d = \frac{G\gamma_w}{1+e}$

$$1.8 = \frac{2.7 \times 1}{1+e}$$

$$\therefore \quad e = 0.5$$

At shrinkage limit, soil is saturated.

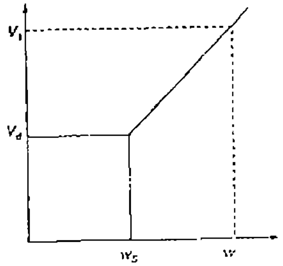
$$\therefore \quad S = 1$$

Using, $S.e = wG$

At shrinkage limit w = shrinkage limit (w_s)

$$\therefore \quad w_s = \frac{Se}{G} = \frac{1 \times 0.5}{2.7} = 0.185$$

Let w be the water content which will fully saturate the sample and cause an increase in volume equal to 10%.



$\therefore \quad V_1 = 1.1V_d = 1.1 \times 200 = 220 \text{ cm}^3$

We know,

Shrinkage limit, $\quad w_s = w_1 - \left(\dfrac{V_1 - V_d}{W_d}\right) \times \gamma_w$

$$0.185 = w - \left(\frac{20}{360}\right) \times 1$$

$\therefore \quad w = 0.24 \text{ or } 24\ \%$

Example 2.34 A soil sample of volume 320 cm³ weighs 600 g. On oven drying, the weight of sample reduced to 90% and volume reduced by 12%. Calculate (i) shrinkage limit (ii) specific gravity of solids (iii) shrinkage ratio

Solution:

Given,

$V_1 = 320 \text{ cm}^3$

$W_1 = 600 \text{ g}$

$V_d = V_2 = 0.92 \times 320 = 294.4 \text{ cm}^3$

$W_d = W_2 = 0.88 \times 600 = 528 \text{ g}$

(i) Shrinkage limit, $\quad w_s = w_1 - \left(\dfrac{V_1 - V_d}{W_d}\right)\gamma_w$

where $\quad w_1 = \dfrac{W_w}{W_s} = \dfrac{W_1 - W_2}{W_2}$

$$\therefore \quad w_s = \left(\frac{W_1 - W_2}{W_2}\right) - \left(\frac{V_1 - V_d}{W_d}\right)\gamma_w$$

$$= \left(\frac{600 - 528}{528}\right) - \left(\frac{320 - 294.4}{528}\right) \times 1$$

$$= 0.087 \text{ or } 8.78\ \%$$

(ii) $$G = \frac{1}{\left(\dfrac{\gamma_w}{\gamma_d} - \dfrac{w_s}{100}\right)}$$

Here, $\quad \gamma_d = \dfrac{W_d}{V_d} = \dfrac{528}{294.4} = 1.79 \text{ g/cc}$

$$\therefore \quad G = \frac{1}{\dfrac{1}{1.79} - 0.087} = 2.12$$

(iii) Shrinkage ratio, $\quad R = \dfrac{\gamma_d}{\gamma_w} = \dfrac{1.79}{1} = 1.79$

Example 2.35 The mass specific gravity of a fully saturated specimen of clay having a water content of 40% is 1.88. On oven drying, the mass specific gravity drops to 1.74. Calculate the specific gravity of clay and its shrinkage limit.

Solution:

For saturated clay,

Mass specific gravity, $\quad G_m = \dfrac{\gamma_t}{\gamma_w} = \dfrac{\gamma_{sat}}{\gamma_w}$

But $\quad \gamma_{sat} = \left(\dfrac{G+e}{1+e}\right)\gamma_w$

$$\therefore \quad G_m = \frac{\dfrac{(G+e)\gamma_w}{(1+e)}}{\gamma_w} = \frac{(G+e)}{1+e}$$

$$\therefore \quad \frac{G+e}{1+e} = 1.88 \qquad \text{...(i)}$$

At saturation, $\quad S = 1 \text{ and } w = 40\ \% \text{ (given)}$

$S.e = w.G$

$$\therefore \quad e = \frac{w.G}{S} = \frac{0.4G}{1} \qquad \text{...(ii)}$$

$e = 0.4\ G$

Substituting value of 'e' in (i), we get

$$\frac{G+0.4G}{1+0.4G} = 1.88$$

$\therefore \quad G = 2.9$

Now, mass sp.gravity for dry soil, $\quad G_m = \dfrac{\gamma_t}{\gamma_w} = \dfrac{\gamma_d}{\gamma_w}$

$$\therefore \quad \frac{\gamma_d}{\gamma_w} = 1.74 \qquad \text{...(iii)}$$

If specific gravity of soil solids is known, then shrinkage limit may be obtained from the relation,

$$G = \frac{1}{\dfrac{\gamma_w}{\gamma_d} - \dfrac{w_s}{100}}$$

$$2.9 = \frac{1}{\left(\dfrac{1}{1.74}\right) - \left(\dfrac{w_s}{100}\right)}$$

$$1.67 = \frac{2.9\,w_s}{100} = 1$$

$$\frac{2.9\,w_s}{100} = 0.67$$

$$w_s = \frac{0.67 \times 100}{2.9} = 23.10\%$$

Example 2.36 The values of liquid limit, plastic limit and shrinkage limit of a soil were reported as below : $w_L = 60\ \%$, $w_P = 30\%$ and $w_S = 20\%$. If a sample of this soil at liquid limit has a volume of 40 cc and its volume measured at shrinkage limit was 23.5 cc, determine the specific gravity of the solids. What is the shrinkage ratio and volumetric shrinkage?

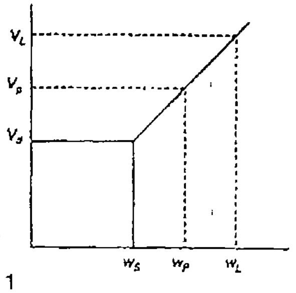
Solution:

Given,

$w_L = 60\ \%\ ,\ V_L = 40\ \text{cc}$

$w_S = 20\ \%,\ V_d = 23.5\ \text{cc}$

$w_P = 30\ \%$



Specific gravity of solids, $G = \dfrac{1}{\dfrac{\gamma_w}{\gamma_d} - \dfrac{w_s}{100}}$

The ratio $\dfrac{\gamma_d}{\gamma_w}$ is known as shrinkage ratio R

$$\therefore \quad G = \frac{1}{\frac{1}{R}-\frac{w_s}{100}} = \frac{1}{\frac{1}{R}-\frac{20}{100}} = \frac{1}{\left(\frac{1}{R}-\frac{1}{5}\right)}$$

$$G = \frac{1}{\frac{1}{R}-0.2} \qquad \text{...(i)}$$

The shrinkage ratio may be given as,

$$R = \frac{\frac{V_1-V_d}{V_d}\times 100}{w_2-w_s}$$

Here $V_1 = V_L$ and $w_2 = w_L$

$$\therefore \quad R = \frac{\frac{V_L-V_d}{V_d}\times 100}{w_L-w_s} = \frac{\frac{40-23.5}{23.5}\times 100}{60-20} = 1.75$$

Substituting value of R in (i), we have

$$G = \frac{1}{\frac{1}{1.75}-0.2} = 2.70$$

Volumetric Shrinkage, $V_s = \dfrac{V_1-V_d}{V_d}\times 100$

Here, $V_1 = V_L$

$$\therefore \quad \frac{V_L-V_d}{V_d}\times 100 = \frac{40-23.5}{23.5}\times 100 \Rightarrow V_s \approx 70.2\ \%$$

Example 2.37 **The natural moisture content of a fully saturated clay is 32.3%. The specific gravity of soil is 2.65. Calculate the percentage decrease to be expected in the unit volume of clay when its moisture content is reduced by evaporation to 16.4%. The shrinkage limit of clay is 23%.**

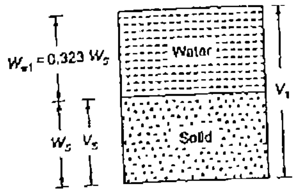
Solution:

Volume of soil at natural condition,

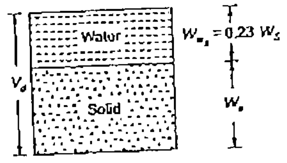
$$V_1 = V_s + V_{w1}$$

$$w = 32.3\ \% \text{ or } 0.323 = \frac{W_{w1}}{W_s}$$

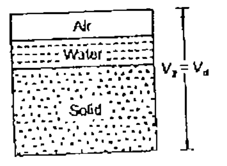
$$\therefore \quad W_{w1} = 0.323\ W_s$$



(i) Original saturated soil



(ii) Soil at shrinkage limit



(iii) Soil at 16.4% moisture content

$$\Rightarrow \quad V_{w1} = \frac{0.323\ W_s}{\gamma_w} = \frac{0.323\ W_s}{1} \qquad [\gamma_w = 1\ \text{g/cc}]$$

Thus $V_1 = V_s + 0.323\ W_s$

Similarly, volume of soil at shrinkage limit

$$V_d = V_s + 0.23\ W_s$$

If water content of soil is reduced to 16.4%, it is below the shrinkage limit. Hence the volume of soil at this water content will remain same as shrinkage limit.

$$\therefore \quad V_2 = V_d$$

$$\%\ \text{Volume reduction} = \frac{V_1-V_2}{V_1}\times 100$$

$$= \frac{(V_s+0.323W_s)-(V_s+0.23W_s)}{V_s+0.323\,W_s}\times 100$$

$$= \frac{9.3W_s}{V_s+0.323W_s} = \frac{9.3W_s}{W_s\left(\frac{V_s}{W_s}+0.323\right)}$$

$$= \frac{9.3}{\frac{V_s}{W_s}+0.323} = \frac{9.3}{\left(\frac{V_s}{V_s\gamma_s}\right)+0.323}$$

$$= \frac{9.3}{\left(\frac{1}{G\gamma_w}\right)+0.323} = \frac{9.3}{\left(\frac{1}{2.65\times 1}+0.323\right)} = 13.28\% \qquad \left[\because G = \frac{\gamma_s}{\gamma_w}\right]$$

Example 2.38 An oven dry sample of clay has volume of 250 cc and weight 410 g. If specific gravity of soil solids is 2.74, determine the void ratio and shrinkage limit. Also find % increase in volume of dry soil, when the sample is fully saturated at a water content 32%.

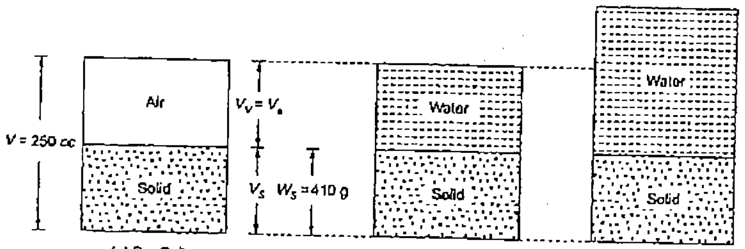
Solution:

Given,

$V_d = 250\ \text{cc}$

$W_d = 410\ \text{g}$

$G = 2.74$



(a) Dry Soil (b) Soil at Shrinkage limit (c) Soil at 32% water content

We know,
$$G = \frac{W_s}{V_s\gamma_w}$$

∴ Volume of solid,
$$V_s = \frac{W_s}{G\gamma_w} = \frac{410}{2.74\times1} = 149.63 \text{ cc}$$

Volume of void,
$$V_v = V - V_s$$

⇒
$$V_v = 250 - 149.63 = 100.37 \text{ cc}$$

∴ Void ratio,
$$e = \frac{V_v}{V_s} = \frac{100.37}{149.63} = 0.67$$

We know,
$$S.e = wG$$

at shrinkage limit,
$$S = 1 \text{ and } w = w_s$$

∴
$$w_s = \frac{e}{G} = \frac{0.67}{2.74} = 0.2448 \text{ or } 24.48\ \%.$$

Volume of soil at 32% moisture,
$$V_2 = V_s + V_w$$
$$w = \frac{W_w}{W_s} = \frac{V_w\times\gamma_w}{W_s}$$

∴
$$V_w = \frac{w\times W_s}{\gamma_w} = \frac{0.32\times410}{1} = 131.2 \text{ cc}$$

∴
$$V_2 = 149.63 + 131.2 = 280.83 \text{ cc}$$

$$\%\text{ increase in volume } = \frac{V_2 - V_1}{V_1}\times100 = \frac{280.83-250}{250}\times100 = 12.33\%$$

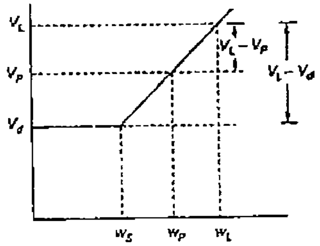
Example 2.39: The plastic limit and liquid limit of a soil are 30% and 42 % respectively. The percent volume change from the liquid limit to the dry state is 35% of the dry volume similarly, the percentage volume change from the plastic limit to the dry state is 22% of the dry volume. Determine the shrinkage limit and shrinkage ratio.

Solution:

Given,
$$w_p = 0.30 \text{ and } w_L = 0.42$$
$$V_L - V_d = 0.35\,V_d$$
$$V_p - V_d = 0.22\,V_d$$

From the figure,

$$\frac{dy}{dx} = \text{slope} = \text{constant}$$



$$\frac{V_L - V_P}{w_L - w_P} = \frac{V_L - V_d}{w_L - w_S}$$
$$\frac{1.35V_d - 1.22V_d}{0.42-0.30} = \frac{1.35V_d - V_d}{0.42 - w_S}$$
$$\frac{0.13V_d}{0.12} = \frac{0.35V_d}{0.42 - w_s}$$
$$(0.42 - w_s) = \frac{0.35\times0.12}{0.13} = 0.323$$
$$w_s = 0.42 - 0.323 = 0.097 \text{ or } 9.7\ \%$$

$$\text{Shrinkage ratio, } R = \frac{\frac{V_1 - V_2}{V_d}\times100}{w_1 - w_2}$$

Here,
$$V_1 = V_L,\ V_2 = V_P$$

and
$$w_1 = w_L,\ w_2 = w_P$$

∴
$$R = \frac{\frac{V_L - V_P}{V_d}\times100}{w_L - w_P} = \frac{\frac{1.35V_d - 1.22V_d}{V_d}}{42-30}\times100 = 1.08$$

2.11.4 Plasticity Index

- It is the range of moisture content over which a soil exhibits plasticity. It is equal to the difference between *LL* and *PL*.

$$I_P = w_L - w_P$$

where,
w_L = Water content at *LL*
w_P = Water content at *PL*

- This is due to presence of clay minerals.
- For coarse grained soils, there is no plastic zone. Hence *LL* coincides with *PL*

∴ $I_P = 0$

- If $PL \ge LL$, then I_P is reported as zero. I_P can never be Negative.

Soil classification based on Plasticity Index

I_P(%)	Soil description
0	Non Plastic
1 to 5	Slight Plastic
5 to 10	Low Plastic
10 to 20	Medium Plastic
20 to 40	Highly plastic
> 40	Very highly plastic

Note: Clay soils possessing high values of liquid limit and plasticity index are referred to as highly plastic or fat clays, and those with low values are described as lean clays.

Plasticity Index of Mixed Soil

Let X_1 % of a soil A having plasticity index I_{P_1}, is mixed with X_2 % of soil B having plasticity index I_{P_2}. Then, I_P of mixed soil will be

$$I_P = \frac{X_1 I_{P_1} + X_2 I_{P_2}}{X_1 + X_2}$$

2.11.5 Shrinkage Index (I_S)

It represents semi-solid state of consistency.

$$I_S = w_P - w_S$$

where, w_P = water content at plastic limit
w_S = water content at shrinkage limit

2.11.6 Consistency Index / Relative Consistency (I_C)

It is defined as a ratio of the difference between the liquid limit and the natural water content of the soil to the plasticity index.

$$I_C = \frac{w_L - w}{I_P} = \frac{w_L - w}{w_L - w_P}$$

where, w_P = water content at PL
w_L = water content at LL
w = natural water content.

% I_c	% Water Content	Consistency
$I_c < 0$	$w > w_L$	Soil is in Liquid State
$0 < I_c < 1$	$w_P < w < w_L$	Soil is in Plastic state
$I_c > 1$	$w < w_P$	Soil is in Solid/semi solid

2.11.7 Liquidity Index (I_L)

It is defined as the ratio of the difference between the natural water content of a soil and its plastic limit to its plasticity index.

$$I_L = \frac{w - w_P}{I_P} = \frac{w - w_P}{w_L - w_P}$$

% I_L	% Water Content	Consistency
$I_L < 0$	$w < w_L$	Solid/semi solid state
$0 < I_L < 1$	$w_P < w < w_L$	Plastic state
$I_L >$	$w > w_L$	Liquid state

Note: Sum of consistency index and liquidity index is always unity.

$$\therefore \quad I_C + I_L = 1$$

2.11.8 Toughness Index (I_t)

- It is defined as the ratio of plasticity index to the flow index.

$$I_t = \frac{I_P}{I_f}$$

- It gives an idea about the shear strength of a soil at plastic limit. For the same value of plasticity index, two soils exhibit different toughness based on flow index.
- For most of the soil :

$$0 < I_t < 3$$

- When $I_t < 1$, the soil is easily crushed (friable) at the plastic limit.

2.12 Sensitivity (S_t)

Consistency of a clay sample is altered even at the same water content on its remoulding. This change in consistency or loss of strength takes place due to the following:

(i) Due to permanent destruction of soil solids on remoulding.
(ii) Due to change in orientation of the water molecules in the adsorbed layer of soil solids.

Sensitivity is defined as the ratio of the unconfined compressive strength of an undisturbed specimen of the soil to the unconfined compressive strength of a specimen of the same soil after remoulding at unaltered water content.

$$S_t = \frac{(q_u)\text{undisturbed}}{(q_u)\text{remoulded}}$$

It represents degree of disturbance achieved on remoulding.

Soil classification based on sensitivity:

Sensitivity	% Water Content	Consistency
1 - 4	Normal	Gravel, Coarse sand
4 - 8	Sensitive	Sand
8 - 15	Extra Sensitive	Flocculent structure soil
> 15	Quick	Fine clay

2.13 Thixotropy

It is that property of soil due to which loss in shear strength due to remoulding at unaltered moisture content may be regained.

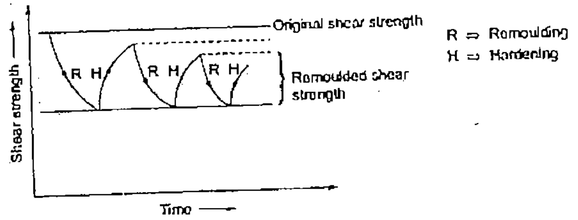


Fig. 2.20

Cohesive soils have greater thixotropy than cohesionless soils.

Higher the sensitivity, larger the thixotropic hardening.

2.14 Unconfined Compressive Strength (q_u)

Defined as the load per unit area at which unconfined cylindrical specimen of standard dimension fails in a simple compression test.

$$q_u = 2\,C_u$$

where, C_u = cohesion of pure clay

q_u is related to consistency of clays.

q_u (kN/m^2)	< 25	25 - 50	50 - 100	100 - 200	200 - 400	> 400
Consistency	Very soft	Soft	Medium	Stiff	Very stiff	Hard

2.15 Activity (A_C)

The behaviour of clay with change in water content is influenced by the presence, magnitude and type of clay minerals.

According to Skempton, Activity is defined as

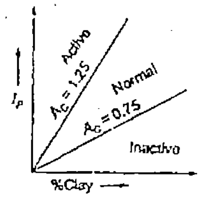
$$A_C = \frac{\%I_P}{\%C}$$

where, I_P = plasticity index

$\%C$ = % of particles which are of clay size i.e. smaller than 2 μm.

Classification of clays, based on Activity

Activity (A_C)	Consistency
< 0.75	Inactive
0.75 < A_C < 1.25	Normal
A_C > 1.25	Active



- Higher activity means soil is more compressible and represents more change in volume due to moisture variation, hence high active soils are not preferred for foundation works.
- Activity of clay minerals :

Type of minerals	Activity (A_C)
Kaolinite	0.38
Illite	0.90
Montmorillonite	7.2

Note: Black cotton soil and bentonite soils have high content of montmorillonite mineral. Hence they are highly active.

2.16 Collapsibility

- Soils which show large decrease in volume with increase in its water content at same pressure are termed as collapsible soil. For example: Loess.
- Collapsibility of the soil is analysed in terms of a parameter termed as collapse potential.
- Collapse potential is defined as the ratio of change in volume of the soil with increase in water content to its original volume.
- Collapse potential can be determined by performing a simple plate load test in the field.

$$\text{Collapse potential, } CP = \frac{\Delta V}{V_0} = \frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0}$$

CP% - Effect on Structure
0 - 1 - No trouble
1 - 5 - Moderate
5 - 10 - Trouble
10 - 20 - Severe trouble
> 20 - Very severe trouble

2.17 Relationship between Atterberg Limits and Engineering Properties

Characterstics	Equal LL (w_L) and Increasing I_P	Equal I_P and Increasing LL (w_L)
Dry Strength	I	D
Toughness	I	D
Compressibility	-	I
Permeability	D	I
Rate of volume change	D	I

where, D = decrease

I = increase

Example 2.40 A soil has liquid limit 40% and plastic limit of 25 %. Determine the toughness index of soil if flow index is 20%.

(a) 1 (b) 0.25
(c) 0.5 (d) 0.75

Ans. (d)

$$I_P = w_L - w_P$$

$$\Rightarrow \quad I_P = 40 - 25 = 15\ \%$$

$$\text{Toughness index, } I_t = \frac{I_P}{I_f}$$

$$\Rightarrow \quad I_t = \frac{15}{20} = 0.75$$

Hence option (d) is correct.

Example 2.41 A sample of the soil has following properties :

Liquid Limit = 45 %
Plastic Limit = 25 %
Shrinkage Limit = 17%
Natural Moisture Content = 30%

The consistency index of the soil is

(a) $\frac{15}{20}$ (b) $\frac{12}{20}$

(c) $\frac{8}{20}$ (d) $\frac{5}{20}$

Ans. (a)

Plasticity index, $I_P = w_L - w_P$

$I_P = 45 - 25 = 20\%$

Consistency index, $I_C = \frac{w_L - w}{I_P} = \frac{45-30}{20} = \frac{15}{20}$

Hence option (a) is correct.

Example 2.42 A soil has following properties :

Liquid Limit = 35
Plastic Limit = 20
Natural Moisture Content = 25%

Match List -I with List-II and select the correct answer using the code given below :

1.	Liquidity index	A.	15
2.	Plasticity index	B.	33
3.	Shrinkage index	C.	10

Codes :

	1	2	3
(a)	A	B	C
(b)	B	A	C
(c)	C	B	A
(d)	B	C	A

Ans. (b)

Plasticity index, $I_P = w_L - w_P$

$\Rightarrow$ $I_P = 35 - 20 = 15\%$

Liquidity index, $I_L = \frac{w - w_P}{I_P}$

$\Rightarrow$ $I_L = \frac{25-20}{15} = 33\%$

Shrinkage index, $I_s = w_P - w_s$

$\Rightarrow$ $I_s = 20 - 10 = 10\%$

Hence option (b) is correct.

Example 2.43

Atterberg's limit	Soil *A*	Soil *B*
Liquid limit	40	0
Plastic limit	15	0

Soil *A* and Soil *B* are mixed to prepare a clayey soil of plasticity index of 12, the percentage of sand in the mix should be

(a) 50 (b) 60
(c) 52 (d) 43

Ans.(c)

$I_{P1} = w_L - w_P = 40 - 15 = 25$

$I_{P2} = 0$

Let percentage of *B* to be mixed is x

$\therefore$ $X_2 = x$ and $X_1 = 100 - x$

$I_{P mix} = \frac{X_1 I_{P1} + X_2 I_{P2}}{X_1 + X_2}$

$12 = \frac{(100-x)\times 25 + x\times 0}{100}$

$1200 = 2500 - 25x$

$x = 52\ \%$

Hence option (c) is correct.

Example 2.44 The liquid limit and plastic limit of sample are 60% and 30% respectively. The percentage of the soil fraction with grain size finer than 0.002 mm is 20. The activity ratio of the soil sample is

(a) 0.50 (b) 1.00
(c) 1.50 (d) 2.00

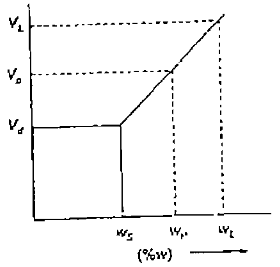
Ans.(c)

$A_C = \frac{I_P}{\%C} = \frac{w_L - w_P}{\%C}$

$\Rightarrow$ $A_C = \frac{60-30}{20} = 1.5$

Hence option (c) is correct.

Example 2.45 The plastic limit of a soil is 24% and its plasticity index is 8%. When the soil is dried from its state at plastic limit, the volume change is 26% of its volume at plastic limit. The corresponding volume change from the liquid limit to dry state is 35% of its volume at liquid limit. Determine the shrinkage limit and the shrinkage ratio.



Solution:

Given, $w_p = 0.24$

$I_p = 0.08$

$I_p = w_L - w_P = 0.08$

$\therefore$ $w_L = 0.08 + 0.24 = 0.32$

$V_p - V_d = 0.26\ V_p$ (given) ...(i)

$V_p - V_d = 0.35\ V_L$ (given) ...(ii)

Substracting (ii) from (i), we get

$V_p - V_L = 0.26\ V_p - 0.35\ V_L$

$1.26\ V_p = 0.65\ V_L$

$V_L = 1.138\ V_p$ ...(iii)

From geometry, $\frac{dy}{dx}$ = slope = constant

$$\frac{w_P - w_S}{V_P - V_d} = \frac{w_L - w_P}{V_L - V_P}$$

$$\frac{0.24 - w_S}{0.26V_P} = \frac{0.32 - 0.24}{1.138V_P - V_P}$$

$$\frac{0.24 - w_S}{0.26} = \frac{0.08}{0.138}$$

or $0.24 - w_S = 0.15$

or $w_S = 0.09 \text{ or } 9\%$

Shrinkage ratio, $$R = \frac{\frac{V_1 - V_2}{V_d} \times 100}{w_1 - w_2} = \frac{\frac{V_L - V_P}{V_d} \times 100}{w_L - w_P}$$

$\Rightarrow$ $$R = \frac{1.38V_P - V_P}{0.74V_P \times (0.32 - 0.24)}$$

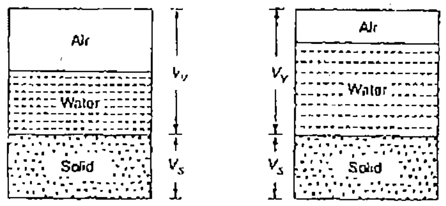
$\Rightarrow$ $$R = \frac{0.138}{0.74 \times 0.08} = 2.33$$

Illustrative Examples

Example 2.46 A clayey soil has saturated moisture content of 15.8%. The specific gravity is 2.72. Its saturation percentage is 70.8%. The soil is allowed to absorb water. After some time, the saturation increased to 90.8%. Find the water content of the soil in the later case.

Solution:

$G = 2.72$, $S = 70.8$, $w = 15.8\%$



(a) case I (b) case II

In Case I,

Void ratio, $$e = \frac{wG}{S} = \frac{0.158 \times 2.72}{0.708} = 0.607$$

Since degree of saturation is within $0 < S < 100\%$. Hence volume of void remain same.

$\therefore$ $e_1 = e_2$

$\therefore$ $$w = \frac{0.908 \times 0.607}{2.72} = 0.2036 \text{ or } 20.36\%$$

Example 2.47 A cohesive soils yields maximum dry density of 1.8 g/cc at an optimum moisture content of 16%. If $G_s = 2.65$, then find the degree of saturation. Also determine the maximum dry density which can be possible to achieved.

Solution:

Given, $\rho_{d,\,max} = 1.8$ g/cc

$w = 16\%$ or 0.16

$G = 2.65$

We know, $$\rho_d = \frac{G\rho_w}{1+e}$$

$\therefore$ $$e = \frac{2.65 \times 1}{1.8} - 1 = 0.472$$

Also, $S.e = wG$

$$S = \frac{wG}{e} = \frac{0.16 \times 2.65}{0.472} = 0.89 \text{ or } 89\%$$

Theoritical maximum dry density will be achieved when all the air present in the voids escaped out. i.e. all voids are just filled by water only.

$\therefore$ For condition of theoritical maximum dry density,

$S = 100\%$ or 1

$S.e = w.G$

$$e = \frac{wG}{S} = \frac{0.16 \times 2.65}{1} = 0.424$$

$\therefore$ Theoritical maximum density, $$\rho_{d.max} = \frac{G\rho_w}{1+e} = \frac{2.65 \times 1}{1+0.424} = 1.86 \text{ g/cc}$$

Example 2.48 You are a project engineer on a large dam project that has a volume of 800,000 m³ of selected fill, compacted such that the final void ratio in the dam is 0.80. The project manager delegates to you the important decision of buying the earth fill from one of the three suppliers. Which one of the three suppliers is the most economical and how much will you save.

Supplier *A* sells fill at 5 ₹/m³ with $e = 1.50$
Supplier *B* sells fill at 10 ₹/m³ with $e = 0.20$
Supplier *C* sells fill at 12 ₹/m³ with $e = 1.60$

Solution:

Without considering void ratio, it would appear that supplier *A* is cheaper than *B* by 5 ₹/m³.

Volume of solid needed for dam site, $$V_s = \frac{V}{1+e} = \frac{800,000}{1+0.80} = 444,444 \text{ m}^3$$

Volume of soil required to be taken out from suppliers,

Supplier *A*, $V_A = V_s(1+e) = 4444.44\ (1 + 1.50) = 1,111,110 \text{ m}^3$

Supplier *B*, $V_B = V_s(1+e) = 444.444\ (1 + 0.2) = 544,\ 442 \text{ m}^3$

Supplier *C*, $V_C = V_s(1+e) = 444,\ 444\ (1 + 1.6) = 1155,\ 555 \text{ m}^3$

Cost of bills,

Supplier *A*, $A = 1,111,110 \times 5 = 5,555,550$ ₹

Supplier *B*, $B = 533,332 \times 10 = 5,333,320$ ₹

Supplier C, C = 1155,555 × 12 = ₹ 13,866,660

Therefore supplier B is more economical, and we save

₹ = 5,555,550 − 5,333,320 = 222,230

Example 2.49 **The fines fraction of a soil to be used for a highway near Hapur was subjected to a hydrometer analysis be placing 25 g of dry soil in 100 ml solution of water (μ = 0.01 poise at 20°c). The specific gravity of the solid was 2.65.**

(a) Estimate the maximum diameter D of the particle found at a depth of 5 cm after a sedimentation of 4 hour has elapsed, if the solution's concentration has reduced to 2 g/lit at the level.

(b) What % of the sample would have a diameter smaller than D?

Solution:

Given, $W_1 = 25$ g, $V = 1000$ ml or 1000 cc

t = 4 hrs = 14,400 sec, H_e = 5 cm, μ = 0.01 Poise = 0.001 Ns/m²

(a) Using Stoke's law,

$$V = \frac{\gamma_w(G-1)}{18\mu}D^2 = \frac{H_e}{t}$$

$$\therefore \quad D = \sqrt{\frac{18\mu H_e}{\gamma_w(G-1)t}} = \sqrt{\frac{18\times0.001\times5\times10^{-2}}{9.81\times(2.65-1)\times14400}}$$

$$= 6.2\times10^{-5}\text{ m} = 0.062\text{ mm}$$

(b) Concentration of solution $= \frac{2g}{l}$ = 2g soil solids in 1 litre

unit weight of solution after 4 hrs,

$$\gamma = \frac{\text{weight of solution}}{\text{volume of solution}} = \frac{W_S+W_w}{V} = \frac{W_s+(V-V_S)\gamma_W}{V}$$

$$\gamma = \frac{2g+\left[V-\frac{W_S}{G\gamma_w}\right]\times\gamma_w}{V} \qquad \left[\because G=\frac{W_S}{V_S\gamma_w}\right]$$

$$\gamma = \frac{2+\left[1000-\frac{2}{2.65\times1}\right]\times1}{1000} = 1.001\text{ g/cc}$$

But $$\gamma = 1+\frac{R_H}{1000}$$

$$\therefore \quad R_H = 1.245$$

% finer than particle of size 0.0615 m

$$\% = \frac{G}{G-1}\gamma_w\left(\frac{V}{W_1}\right)\left(\frac{R_H}{10}\right)\%$$

$$= \frac{2.65}{2.65-1}\times1\times1\left(\frac{100}{25}\right)\times\left(\frac{1.245}{10}\right) = 8\%$$

Summary

- Soil mass in general is a three phase system composed of solid, liquid and gas.
- Fine grained soils have higher values of natural moisture content as compared to coarse grained soils.
- Void ratio of fine grained soils are generally higher than those of coarse grained soils.
- The unit weight/density of soil (dry, saturated or submerged) is a function of void ratio and moisture content.
- Consistency limits or Atterberg limits viz. liquid limit, plastic limit, and shrinkage limit are the water content at the change of states from liquid to plastic, plastic to semi-solid and semisolid to solid state respectively.
- Plasticity index refers to the range over which soil exhibit plasticity.
- The sum of consistency index and liquidity index is always unity.
- Thixotropy of soil is that property of soil due to which soil regains some part of its lost strength during remoulding.

Objective Brain Teasers

Q.1 In its natural condition, a soil sample has a mass of 2290 gm and a volume of 1.15 × 10^{-3} m³. After being completely dried in an oven, the mass of the sample is 2035 gm. The value of G for the soil is 2.68. Match List-I (Property) with List-II (Values) and select the correct answer using the codes given below:

List-I	List-II
A. Void ratio	1. 0.510
B. Porosity	2. 0.337
C. Degree of saturation	3. 0.657
D. Air content	4. 0.343

Codes:

	A	B	C	D
(a)	1	3	2	4
(b)	1	2	3	4
(c)	2	4	3	1
(d)	4	3	2	1

Q.2 A mass of soil coated with a thin layer of paraffin wax weighs 700 gm and soil alone weighs 650 gm. When soil sample is immersed in water it displaces 400 mL of water. The specific gravity of soil is 2.65 and that of wax is 0.9 and water content is 20%, ρ_w = 1000 kg/m³. Void ratio of soil is

(a) 0.81 (b) 0.59

(c) 0.73 (d) 0.69

Q.3 Sieve analysis is done on a soil sample and following observation were made:

Size of sieve	% Retained
4.75 mm	36
75 micron	90

Size of particle for which 60% particles are finer = 5 mm

Size of particle for which 30% particles are finer = 3 mm

Size of particle for which 10% particles are finer = 1 mm

On further testing the finer particles, it was found that:

Liquid limit = 50%

Plastic limit = 35%

Based on the above information, according to Indian soil classification system, the soil is

(a) SW − SC (b) SP − SM

(c) GW − GC (d) None of these

Answers

1. (b) 2. (d) 3. (d)

Hints and Explanations:

1. (b)

Mass of the soil solid,

$$m_s = 2290 \text{ gm}$$

Mass of dry soil

$$m_d = 2035 \text{ gm}$$

Specific gravity

$$G = 2.68$$

Volume. $V = 1.15 \times 10^{-3} \text{ m}^3$

∴ Density of dry soil

$$\gamma_d = \frac{2035 \times 10^{-3}}{1.15 \times 10^{-3}}$$
$$= 1769.565 \text{ kg/m}^3$$

$$\because \quad \gamma_d = \frac{G\gamma_w}{1+e}$$

$$\Rightarrow \quad 1769.565 = \frac{2.66 \times 1000}{1+e}$$

$$\Rightarrow \quad e = 0.51$$

∴ Void ratio, $e = 0.51$

Porosity $n = \frac{e}{1+e} = 0.338$

Water content

$$w = \frac{m_w}{m_d}$$
$$= \frac{2290 - 2035}{2035} = 0.125$$

$$m_d = \frac{m}{1+w}$$

$$\Rightarrow \quad 2035 = \frac{2290}{1+w}$$

$$\Rightarrow \quad w = 0.125$$

$$\therefore \quad wG = eS$$

$$\Rightarrow \frac{0.125 \times 2.68}{0.51} = S$$

$$\Rightarrow \quad S = 0.657$$

∴ Degree of saturation = 0.657

Air content, $= 1 - S$
$= 1 - 0.657$
$= 0.343$

2. (d)

$$M_{wax} = 700 - 650 = 50 \text{ gm}$$
$$\gamma_{wax} = 0.9 \times 1 = 0.9 \text{ gm/cc}$$
$$V_w = V_1 + V_{wax}$$
$$V_1 = V_w - V_{wax} = 400 - \frac{M_{wax}}{\gamma_{wax}}$$
$$V_1 = 400 - \frac{50}{0.9} = 344.4 \text{ ml.}$$
$$\rho = \frac{M}{V_1} = \frac{650}{344.4} = 1.89 \text{ gm/cc}$$
$$\rho_d = \frac{\rho}{1+w} = \frac{1.89}{1.20} = 1.57 \text{ gm/cc}$$
$$\rho_d = \frac{G\rho_w}{1+e}$$
$$1.57 = \frac{2.65 \times 1}{1+e}$$
$$e = 0.69$$

3. (d)

% retained on 75 micron sieve is 90%

∴ Soil is coarse

% Gravel = 36

% Sand = 90 − 36 = 54

More than half of coarse fraction is sand

$$D_{60} = 5 \text{ mm}$$
$$D_{30} = 3 \text{ mm}$$
$$D_{10} = 1 \text{ mm}$$
$$C_u = \frac{D_{60}}{D_{10}} = \frac{5}{1} < 6$$

Finer analysis: It is poorly graded

$$I_p = \text{Liquid limit} - \text{Plastic limit}$$
$$= 50 - 35 = 15\%$$

A-line, $I_p = 0.73\,(w_L - 20)$
$= 0.73\,(50 - 20) = 21.9$

This will lie above A-line, therefore, it is clay.

Soil is SP - SC.

Student's Assignments

Q.1 A soil sample with specific gravity of solids 2.70 has a mass specific gravity of 1.84. Assuming the soil to be perfectly dry, determine the void ratio.

[Ans. 0.47]

Q.2 An oven dry soil sample of volume 250 cc weight 430 g. If the specific gravity of solids is 2.70, what is the water content when the soil becomes fully saturated without any change in its volume? What will be the water content which will fully saturate the sample and also cause an increase in volume equal to 10% of the original dry volume? Use phase diagram.

[Ans. 21.1%, 26.9%]

Q.3 An airport runway fill needs 600,000 m³ of soil compacted to a void ratio of 0.75. There are two borrow pits A and B from where the required soil can be taken and transported to the site.

Borrow pit	In situ void ratio	Transportation cost
A	0.80	₹ 10/m³
B	1.70	₹ 5/m³

Which of the two borrow pits would be more economical?

[Ans. ₹ 6171430, ₹ 4628570]

Q.4 An undisturbed sample of clay brought from the field was noted to have a volume of 18.0 cc and weight of 30.8 g. On over drying, the weight of the sample was reduced to 20.5 g. The volume of dried sample as obtained by displacement of mercury was 12.5 cc. Calculate the shrinkage limit and the specific gravity of solids. What is the shrinkage ratio?

[Ans. 1.64]

Q.5 A natural soil deposit has a bulk unit weight of 18 kN/m³ and a water content of 5%. Calculate the amount of water in litres required to be added to 1 cu m of soil to raise the water content to 15%. Assume the void ratio to remain constant. What will be the degree of saturation? Take $G_s = 2.60$.

[Ans. 174 liters; S = 79.6%]

Q.6 The dry unit weight of a 4 m thick stratum of coarse sand is 15 t/m³. Calculate the quantity of cement grout slurry required per meter run for providing a 2 m thick grout curtain through this stratum. G_s of sand grains = 2.68.

[Ans. 3.43 m³]

Q.7 A sample of soil with a liquid limit of 72.8% was found to have a liquidity index of 1.21 and water content of 81.3%. What are its plastic limit and plasticity index? Comment on the consistency of the soil.

[Ans. 32.3%, 40.5%]

Q.8 A soil has a liquid limit of 50% and plasticity index of 20. When the soil at its liquid limit was dried, the percentage decrease in volume was 40% of its dry volume. When it was dried from its plastic limit, the percentage decrease in volume as 20% of its dry volume. Determine the shrinkage limit of the soil and its shrinkage ratio.

[Ans. 10%, 1.0]

Q.9 A sample of sand from a natural deposit has a porosity of 35%. For a volume of 495 cc, the dry weight in the densest and loosest states are 950 and 700 g, respectively. Compute the relative density of sand assuming the specific gravity of solids to be 2.65.

[Ans. 67.9%]

■■■■

CHAPTER 03

Identification and Classification of Soils

3.1 Introduction

Soil classification is the arrangement of soils into different groups such that the soils in a particular group have similar behaviour under given set of physical conditions. Generally soil classification is done on the basis of two criteria, viz.

(a) Grain size distribution
(b) Plasticity of soil

3.2 Field Identification of Soils

Broadly soil can be categorized as coarse grained (cohesionless) and fine grained soils (cohesive).

(a) Coarse Grained Soils

Coarse grained soils can be easily identified by the naked eye on the basis of grain size. The major coarse grained soils are gravel and sand.

- Individual soil particles larger than 4.75 mm and smaller than 80 mm are called 'gravel'.
- Soil particles ranging in size from 0.075 mm to 4.75 mm are called 'sand'.

(b) Fine Grained Soils

Silt and clays are known as fine grained soils. Fine soils can be organic or inorganic.

Inorganic Soils: Field identification of inorganic soils can be done by following tests

(i) Dilatancy test or shaking test
(ii) Dry strength test
(iii) Dispersion test
(iv) Toughness test or Rolling test

(i) Dilatancy Test (Dilatancy → Reaction to Shaking): A part of the material is shaken after placing it on the palm. If it is silt, water comes to the surface and gives it a shining appearance. If it is kneaded, the moisture will re-enter into the soil and the shining disappears.

If it is clay, the water cannot move easily and hence it continues to look dark. If it is a mixture of silt and clay, the relative speed with which the shine appears may give rough indication of the amount of silt present. This test is also known as 'Shaking test'.

(ii) Dry Strength Test (Dry strength → Crushing Resistance): A small ball or briquette of size 3 mm is prepared and kept for drying. If dried ball can be easily broken, the material is silt. If it is clay, it will require effort to break. Also one can dust off loose material form the surface of briquette if it is silt. When moist soil is pressed between fingers, clay gives a soapy touch; it also sticks and dried slowly which cannot be dusted easily.

(iii) Dispersion Test: A spoonful of soil is poured in a jar of water. If it is silt, the particles will settle in about 15 minutes to one hour. If it is clay, it will form a turbid suspension, which will remain as such for hours and even for days.

(iv) Toughness Test or Thread Test (Toughness → Consistency near plastic limit): A thread is attempted to be made out of a moist soil sample with a diameter of about 3 mm. If the material is silt, it is not possible to make such thread without surface cracks and crumbling.

If it is clay, such a thread can be made even to a length of about 30 cm and supported by its own weight when held at the ends.

This test is also called 'Rolling test'.

Organic Soils:

- Fresh, wet organic soils usually have a distinctive odour of decomposed organic matter, which can be more easily detected on heating the wet sample.
- Another distinctive feature of such soils is the dark colour.

3.3 Engineering Classification of Soils

Several classification systems were developed by various organizations for their specific purpose.

Some important classification system are as follows:

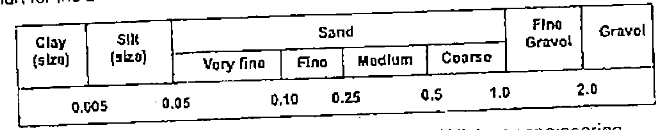
- Classification based on grain size
- Textural classification
- Unified soil classification system (USCS)
- American association of state highway and transport officials (AASHTO) system
- Indian standard soil classification system (ISSCS)

3.3.1 Texural Classification (more suitable for coarse grained soils)

- In this system, soil fractions as per the US Bureau of soils and chemistry system are used.
- According to this classification

Gravel : > 1.00 mm
Sand : 0.05 mm - 1.00 mm
Silt : 0.005 mm - 0.05 mm
Clay : < 0.005 mm

- A triangular chart has been developed using grain size distribution. First of all grain size distribution of the soil is found and % fractions are determined with the well known percentages of sand, silt and clay, a point is located in the triangular chart as shown in figure, specified term designated in the chart for the area where the point falls is taken as classification of the soil.



- This classification system widely used in Agriculture and Highway engineering
- This classification depends on the grain size distribution only.

3.3.2 The Unified Soil Classification System (USCS)

- This system (USCS), originally developed by A. Casagrande.
- According to USCS, the coarse grained soils are classified on the basis of their grain size distribution and fine grained soils (whose behaviour is controlled by plasticity) on the basis of their plasticity characteristics.
- Soils are classified into four major groups:
  - (a) Coarse grained
  - (b) Fine grained
  - (c) Organic soils
  - (d) Peat

Note: USCS is adopted by IS classifications with slight modifications.

3.3.3 'AASHTO' Soil Classification System

- According to AASHTO system, the soils are classified into eight groups: *A*-1 through *A*-7 with an additional group *A*-8 for peat or muck.
- This system includes several sub groups. Soil within each group are evaluated according to the group index calculated from empirical formula.

$$\text{Group Index, } GI = 0.2a + 0.005ac + 0.01bd$$

where,

a = that part of the percentage passing the 75 μ sieve greater than 35 and not exceeding 75.

b = that part of the percent passing the 75 μ sieve greater than 15 and not exceeding 55.

c = that part of liquid limit greater than 40 and not greater than 60.

d = that part of plasticity index greater than 10 and not exceeding 30.

3.3.4 Indian Standard Soil Classification System (ISSCS)

The Indian soil classification (IS: 1498, 1970) is basically the same as that of USCS with the slight modification that the fine grained soil have been subdivided into three subgroup of low, medium and high compressibility as against only two in the USCS.

- In this system, coarse grain soils are classified on the basis of grain size distribution and fine soils on the basis of plasticity.
- Broadly soils are divided into four major groups
  1. Coarse grained soils : gravel (G) and sand (S)
  2. Fine grained soils : silt (S) and clay (C)
  3. Organic soils (O)
  4. Peat (Pt)

Table 3.1: I.S. Classification (grain size distribution)

		Soils						
Boulder (mm)	Cobble (mm)	Coarse grained soil					Fine Grained Sand	
		Gravel		Sand			Silt (mm)	Clay (mm)
		Coarse (mm)	Fine (mm)	Coarse (mm)	Medium (mm)	Fine (mm)		
> 300	300 - 80	20 - 80	4.75 - 20	2 - 4.75	0.425 - 2.0	0.075 - 0.425	0.002 - 0.075	< 0.002

Table 3.2: Basic Soil Components

Sl. No.	Soil component	Symbol	Particle-size range and description
	Boulders	None	Rounded to angular, bulky, hard, rock particle; average diameter more than 300 mm
	Cobbles	None	Rounded to angular, bulky, hard, rock particle; average diameter smaller than 300 mm but retained on 80 mm IS sieve
(1)	Coarse-grained soils Gravel	G	Rounded to angular, bulky, hard, rock particle; passing 80 mm IS sieve but retained on 4.75 mm IS sieve Coarse: 80 mm to 20 mm Fine: 20 mm to 4.75 mm
	Sand	*S*	Rounded to angular, bulky, hard, rock particle; passing 4.75 mm IS sieve but retained on 75 micron Coarse: 4.75 mm to 2.0 mm Medium: 2.0 mm to 425 micron Fine: 425 micron to 75 micron
(2)	Fine-grained soils Silt	*M*	Particles smaller than 75 micron; identified by behaviour, that is, slightly plastic or non-plastic regardless of moisture and exhibits little or no strength when air dried.
	Clay	*C*	Particles smaller then 75 micron; identified by behaviour, that is, it can be made to exhibit plastic properties within a certain range of moisture and exhibits considerable strength when air dried.
(3)	Organic matter	*O*	Organic matter in various sizes and stage of decomposition (no specific grain size)
(4)	Peat	*Pt*	Fibrous, spongy (no specific grain size)

3.4 Classification of Coarse Grained Soil

Classification of coarse grained soil is done on the following basis

1. Particle size
2. Gradation characteristics i.e. C_u and C_c
3. % fineness (% fraction which pass through 75 μ sieve)

- Coarse grained soil are those having 50% or more retained on the 0.075 mm sieve (75 micron).
- Further, the coarse grained soils are designated as gravel (G) if 50% or more of the coarse fraction is retained on the 4.75 mm sieve; otherwise, they are designated as sand (S).

Table 3.3 : Prefix and Suffix of ISSCS

Soil type	Prefix	Subgroup	Suffix
Gravel	G	Well graded	*W*
Sand	S	Poorly graded	*P*
Silt	*M*	Silty	*M*
Clay	C	Clayey	*C*
Organic	O	w_L < 35 percent	*L*
		35 < w_L < 50	*I*
Peat	Pt	50 < w_L	*H*

NOTE

- Well graded soils (W) have wide range of particle size where as poorly graded soil (P) has most of the particle about same size i.e. either excess or a deficiency of certain particle sizes.
- To check, whether a soil is well graded or poorly graded grain size distribution curve is plotted and coefficient of uniformity (C_u) and coefficient of curvature (C_c) are found.

On the basis of % fineness, coarse grained soils can be further classified as:

CASE-1 : When fineness is less than 5% by weight.

Gravels: more than 50% of coarse fraction retained on 4.75 mm sieve

(i) GW ⇒ well graded gravel.
$C_u > 4, 1 \le C_c \le 3$ and fineness < 5%

(ii) GP ⇒ poorly graded gravel.
Above values of C_u and C_c are not satisfied.

Sand: more than 50% of coarse fraction pass through 4.75 mm sieve

(i) SW ⇒ well graded sand.
$C_u > 6, 1 \le C_c \le 3$ and fineness < 5%

(ii) SP ⇒ poorly graded sand.
Above values of C_u and C_c are not satisfied

CASE-2 : When fineness is between 5-12%

This is known as borderline case and dual symbols are used. First part of dual symbol represent gradation and second part represents nature of fines i.e. clay or silt.

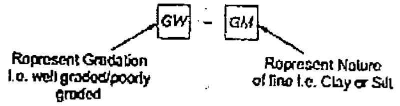


Fig.3.1

Gravel:

(i) GW – GC ⇒ well graded gravel containing clay as fine
$C_u > 4, 1 \le C_c \le 3$
clay fraction > silt fraction

(ii) GP – GC ⇒ poorly graded gravel containing clay as fine
Above values of C_u and C_c are not satisfied
clay fraction > silt fraction

(iii) GW – GM ⇒ well graded gravel containing silt as fine
$C_u > 4, 1 \le C_c \le 3$
silt fraction > clay fraction

(iv) GP – GM ⇒ poorly graded gravel containing silt as fine
above values of C_u and C_c are not satisfied
silt fraction > clay fraction

Sand: more than 50% of coarse fraction pass through 4.75 mm sieve

(i) SW – SC ⇒ well graded sand containing clay as fine
$C_u > 6, 1 \le C_c \le 3$
clay fraction > silt fraction

(ii) SP – SC ⇒ poorly graded sand containing clay as fine
values of C_u and C_c are not satisfied
clay fraction > silt fraction

(iii) SW – SM ⇒ well graded sand containing silt as fine
$C_u > 6, 1 \le C_c \le 3$
silt fraction > clay fraction

(iv) SP – SM ⇒ poorly graded sand containing silt as fine
values of C_u and C_c are not satisfied
silt fraction > clay fraction

CASE-3 : When fineness is more than 12%

In this case soil is classified according to I.S. plasticity chart (%I_p)

Gravel:

(i) GC ⇒ Clayey gravel
% fineness > 12 and $I_p > 7\%$
clay fraction > silt fraction [$\because I_p > 7\%$]

(ii) GM ⇒ Silty gravel
% fineness > 12 and $I_p < 4\%$
silt fraction > clay fraction [$\because I_p < 4\%$]

Sand:

(i) SC ⇒ Clayey sand
% fineness > 12 and $I_p > 7\%$
clay fraction > silt fraction [$\because I_p > 7\%$]

(ii) SM ⇒ Silty sand
% fineness > 12 and $I_p < 4\%$
silt fraction > clay fraction [$\because I_p < 4\%$]

NOTE: If plasticity index is between 4% - 7% then dual symbols are used.

Example3.1 From a particle-size distribution curve of a sandy soil, the following data is obtained:

Size of particle (mm)	Percentage finer
0.46	60
0.33	30
0.21	10

Determine the uniformity coefficient and coefficient of curvature. Is this soil well graded or poorly graded?

Solution:

Given $D_{60} = 0.48$ mm, $D_{30} = 0.33$ mm and $D_{10} = 0.21$ mm

Coefficient of curvature, $C_u = \dfrac{D_{60}}{D_{10}}$

$$\Rightarrow \quad C_u = \frac{0.48}{0.21} = 2.28$$

and coefficient of curvature, $C_c = \dfrac{(D_{30})^2}{D_{60} \times D_{10}}$

$$C_c = \frac{(0.33)^2}{0.48 \times 0.21} = 1.08$$

For well graded sand, $C_u > 6$ and $1 \le C_c \le 3$

So it is a poorly graded sand.

Example 3.2

Sieve Size	4.75 mm	75 Micron
% Finer	80	7

For the above soil sample, the size of aggregates vary linearly and it is found that particles finer than 75 micron are non-plastic. According to Indian standard classification system, the soil sample is

(a) *SP-SM*
(b) *SW-SM*
(c) *SP-SC*
(d) *SW-SC*

Ans. (b)

It is a coarse soil, as only 7% are finer than 75 μ sieve. Also it is a sandy soil because more than 50% of the coarse fraction passes through 4.75 mm sieve.

Since size of aggregates varies linearly.

$$D_{60}: \quad \frac{80-7}{4.75-0.075} = \frac{80-60}{4.75-D_{60}}$$

$$\Rightarrow \quad D_{60} = 3.47 \text{ mm}$$

$$D_{30}: \quad \frac{80-7}{4.75-0.075} = \frac{80-60}{4.75-D_{30}}$$

$$\Rightarrow \quad D_{30} = 1.548 \text{ mm}$$

similarly,
$$D_{10} = 0.267 \text{ mm}$$

$$\therefore \quad C_u = \frac{D_{60}}{D_{10}} = 12.99 > 6$$

and
$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}} = 2.59$$

It means, $1 \le C_c \le 3$

Thus soil is well graded, also the finer particles are non-plastic, so it is silt.

Hence option (b) is correct.

Example 3.3 Laboratory sieve analysis was carried out on a soil sample using a complete set of standard IS-Sieves. Out of 500 g of soil used in the test, 200 g was retained on IS 600 μ sieve, 250 g was retained on IS 500 μ sieve and the remaining 50 g was retaining on IS 425 μ sieve. Find the coefficient of uniformly. Also classify the soil.

Solution:

S.N.	Sieve size	Weight retained (g)	Cum-weight retaining (g)	Cum. % Retained	% Finer % N
1	600 μ	200	200	40	60
2	500 μ	250	450	90	10
3	425 μ	50	500	100	0

$$\therefore \quad D_{60} = 600\,\mu \quad \text{and} \quad D_{10} = 500\,\mu$$

Coefficient of uniformity, $C_u = \frac{D_{60}}{D_{10}}$

$$\Rightarrow \quad C_u = \frac{600\,\mu}{500\,\mu} = 1.2$$

More than 50% of the soil pass through 600 μ sieve, it means that greater percentage of the soil will pass through 4.75 mm sieve. Hence the soil is definitely sandy soil.

For well graded sand

$C_u > 6$ and $1 \le C_c \le 3$

Here, $C_u = 1.2$ thus the soil is poorly graded sand.

Example 3.4 Classify the soil for data given:

Sieve size (mm)	Weight retained (g)
4.75	20
0.075	730

1000 g of soil was used.
Liquid Limit = 40%
Plastic Limit = 18%
The soil classification is

(a) GM
(b) SM
(c) GC
(d) ML-MI

Ans. (a)

S.N.	Sieve size (mm)	Weight retained (g)	Cum-weight retained (g)	Cum. % retained	% Finer % N
1	4.75	20	20	2	98
2	0.075	730	750	75	25

Since 98% of soil pass through 4.75 mm IS sieve, and 75% are retained on 75 μ, given soil is sand.

Also, 25% soil is pass through 0.75 mm sieve.

$\therefore$ fineness = 25%

$W_L = 40\%,\ W_P = 18\%$

$\therefore$ $I_p = 22\% > 7\%$

here fineness > 12% and $I_p > 7\%$

$\therefore$ soil is clayey sand (SC)

3.5 Classification of Fine Grained Soil

- In ISSCS, fine grained soils are classified on the basic of plasticity chart (I_p) and compressibility (w_L)
- Generally soils are considered as fine soils, when 50% or more of the total material by weight pass 75 μ sieve.
- *LL* (w_L) and *PL* (w_p) are determined for 425 μ sieve fraction and corresponding plasticity index is found out.

$$I_p = w_L - w_p$$

CASE-1: Low Plastic Soil (Low Compressibility) (LL < 3.5%)

CL → Low plastic inorganic clay
ML → Low plastic silt
OL → Low plastic organic clay

CASE-2: Medium Plastic Soil (Medium Compressibility) (35% < LL< 50%)

CI → Medium plastic inorganic clay

MI → Medium plastic silt

OI → Medium plastic organic clay

CASE-3: Highly Plastic Soils (High compressibility) (*LL* > 50%)

CH → High plastic inorganic clay

MH → High plastic silt

OH → High plastic organic

NOTE Organic and inorganic soils are plotted in same zone in plasticity chart which are distinguished by odour and colour or liquid limit test on oven dry sample. If LL of oven dry sample is less than the three fourth of in-situ soil sample then soil is organic otherwise inorganic.

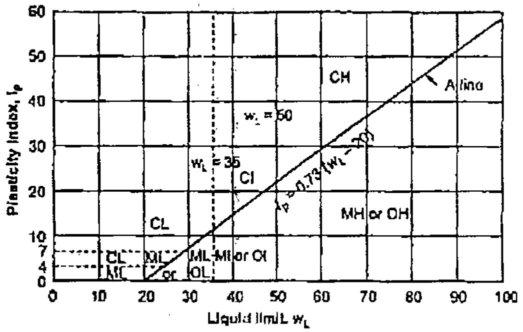


Fig.3.2 Plasticity chart as per Indian Standard Soil Classification System

The above soil classification is based on a line called A-line, which is a boundary representing relationship between plasticity index (I_P) and LL (W_L).

- If I_P of soil > I_P of *A*-line
  the soil will lie above *A*-line and it will be clay(*C*)
- If I_P of soil < I_P of *A*-line
  the soil will lie below *A*-line and it may be either silt (*M*) or organic clay (*OI*)
- The I_P of *A*-line is given by

$$I_P = 0.73\,(W_L - 20)$$

where W_L = liquid limit

- *U*-line represent upper boundary beyond which no soil will lie. If results are found above U-line then test must be repeated.

$$I_P \text{ of } U\text{-line} = 0.9\,(W_L - 8)$$

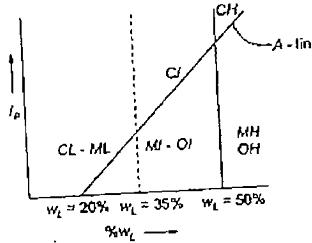
where W_L = liquid limit

- Highly organic soils (eg. Peat) are classified as *Pt*.

Example 3.5 As per the Indian standard soil classification system, a soil sample of silty clay with liquid limit 40% and plasticity of 28% is classified as

(a) *CP*
(b) *CI*
(c) *CL*
(d) *CL – ML*

Solution:



$$I_P \text{ of soil} = 28\%$$

$$I_P \text{ of } A\text{-line} = 0.73\,(W_L - 20)$$

$$= 0.73\,(40 - 28)$$

$$= 8.76$$

$$I_P \text{ of soil} > I_P \text{ of } A\text{-line}$$

∴ It will lie above *A*-line

Also $35 < W_L < 50$

So soil is *CI*.

Example 3.6 A soil has following properties:

LL = 40%; *PL* = 30%;

% passing through 4.75 mm sieve = 65% and % passing through 75 μ sieve = 58%.

The soil can be classified as

(a) *SC*
(b) *CL*
(c) *MI*
(d) *MH*

Ans. (c)

$$\text{Gravel fraction} = \%\text{ retained on 4.75 mm sieve} = 65\%$$

$$\text{Coarse fraction} = \%\text{ retained on } 75\,\mu\text{ sieve} = 58\%$$

$$\text{Sand fraction} = 65 - 58 = 7\%$$

Since more than 50% soil pass through 75 μ sieve, soil is fine grained.

$$I_P \text{ of } A\text{-line} = 0.73(W_L - 20)$$

$$I_{P,A} = 0.73(40 - 20) = 14.6\%$$

∴ $$PI \text{ of soil} = W_L - W_P = 40 - 30$$

$$PI = 10\% \text{ is less than } I_{P,A}$$

Thus silty soil, since $35 < W_L < 50$

∴ Soil may be *MI* or *OI*

Example 3.7 A soil has following characteristics

% passing 75 μ sieve = 62%; Liquid limit = 40%

Plasticity index, I_P = 10%; Liquid limit of oven dry sample = 25%

Classify the soil according to ISSCS system.

(a) *MI*
(b) *OI*
(c) *CH*
(d) *CI*

Ans. (b)

More than 50% of soil pass through 75 μ sieve. So soil is fine grained.

$$I_P \text{ of } A\text{-line} = 0.73(W_L - 20) = 0.73\,(40 - 20) = 14.6\%$$

$$I_P \text{ of soil} = 10\%$$

I_P of soil < I_P of *A*-line. Hence soil will lie below *A*-line

Also, $35 < W_L < 50$, so may be *MI* or *OI*

Since liquid limit of oven dry soil is 25% which is less than $\frac{3}{4}$ of *LL* of soil.

Therefore soil is *OI* (medium plastic organic clay)

Example 3.8 : A soil has the following characteristics:

1. Percentage of soil passing 75 μm sieve = 20
2. Percentage of fraction retained by 4.75 mm sieve = 60
3. Liquid limit = 35
4. Plastic limit = 20

Classify the given soil as per IS-soil classification system.

Solution:

% of soil retained on the 75 μ sieve = 80

It is greater than 50% hence it is a coarse grained soil.

Since gravel fraction (i.e. 60%) is greater than 50, Hence large proportion of the coarse grained soil is gravel, the soil is gravel.

Fineness = % fraction which pass through 75 μ sieve
= 20% (which is greater than 12%)

In this case, soil is classified according to IS-plasticity chart

$$I_P = W_L - W_P = 35 - 20 = 15 > 7\%$$

It means clay fraction > silt fraction

Hence the soil is classified as GC.

Example 3.9 : The following test results were obtained on a soil sample:

Percentage passing through 4.75 mm IS sieve = 98.5%

Percentage passing through 75 μ IS sieve = 35%

$D_{60} = 0.22$, $D_{30} = 0.19$, $D_{10} = 0.16$

Liquid limit = 22%

Plasticity limit = 19%

Classify the soil by IS classification.

Solution:

Percentage of soil retained on the 75 μ sieve = 65

It is greater than 50%. Hence it is a coarse grained soil.

Since gravel fraction (i.e. 1.5%) is less than 50, large proportion of the coarse grained soil is sand

Fineness = % fraction which pass through 75 μ sieve
= 35%

It means fines is greater than 12%

In this case, soil is classified as per IS-classification

$$I_p = W_L - W_P = 22 - 19 = 3\%$$

which is less than 4%

It means silt fraction > clay fraction

Also,

Coefficient of uniformity, $C_u = \frac{D_{60}}{D_{10}} = \frac{0.22}{0.16} = 1.39 < 6$

and coefficient of curvature, $C_c = \frac{D_{30}^2}{D_{60} \times D_{10}} = \frac{(0.19)^2}{0.22 \times 0.16} = 1.03$

For the soil to be well graded,

$C_u > 6$ and $1 < C_c < 3$

Hence the given soil is classified as poorly graded silty sand i.e. SM

Summary

- Broadly soils are classified as coarse grained (cohessionless) and fine grained (cohesive) soils.
- Gravel and sand are put in the category of coarse soil whereas silt and clays are put in the category of fine soils.
- Generally soil classification is done on the basis of two criteria, viz
  - (a) Grain size distribution
  - (b) Plasticity of soil
- The Indian soil classification system is basically the same as the unified soil classification system but for a slight modification in the plasticity chart.
- For classification of fine soils, organic and inorganic soils are plotted in same zone in plasticity chart, which are distinguished by odour, colour or liquid limit on oven dry sample. If LL of oven dry sample is less then of three fourth of in-situ soil sample, then soil is called organic soil, otherwise inorganic.

Objective Brain Teasers

Q.1 The properties of soil A and soil B are given below:

A	B
LL = 60%	LL = 55%
PL = 35%	PL = 20%
w = 40%	w = 25%
G = 2.65	G = 2.7
S = 80%	S = 100%

Which of the following statements are correct?

(a) A has more clay particles and more dry density than B.

(b) A has more clay particles but less dry density than B.

(c) B has more clay particles and more dry density than A.

(d) B has more clay particles but less dry density than A.

*Q.2 Match List-I (Symbol) with List-II (Soil) and select the correct answer using the codes given below the lists:

List-I	List-II
A. ML	1. Silty sand
B. SM	2. Inorganic silt with large compressibility
C. PI	3. Inorganic silt with small compressibility
D. MH	4. Soil with high organic content with high compressibility

Codes:

	A	B	C	D
(a)	3	2	4	1
(b)	4	1	3	2
(c)	3	1	4	2
(d)	4	2	3	1

Q.3 Match List-I (Soil classification symbol) with List-II (Soil property) and select the correct answer using the codes given below the lists:

List-I	List-II
A. GW	1. Soil having uniformity coefficient > 6

B. SW	2. Soil having uniformity coefficient > 4
C. ML	3. Soil have low plasticity
D. CL	4. Soil have low compressibility

Codes:

	A	B	C	D
(a)	1	2	4	3
(b)	2	1	3	4
(c)	2	1	4	3
(d)	1	2	3	4

Q.4 Match List-I (Soils) with List-II (Group symbols) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Clayey gravel	1. SM
B. Clayey sand	2. OH
C. Organic clay	3. SC
D. Silty sand	4. GC

Codes:

	A	B	C	D
(a)	3	4	2	1
(b)	4	3	1	2
(c)	4	3	2	1
(d)	3	4	1	2

Q.5 A soil mass contains 40% gravel, 50% sand and 10% silt. This soil can be classified as

(a) silty sandy gravel having coefficient of uniformity less than 60.

(b) silty gravelly sand having coefficient of uniformity equal to 10.

(c) gravelly silty sand having coefficient of uniformity greater than 60.

(d) gravelly silty sand and its coefficient of uniformity cannot be determined.

Q.6. In a soil specimen, 70% of particles are passing through 4.75 mm IS sieve and 40% of particles are passing through 75 μ IS sieve. Its uniformity coefficient is 8 and coefficient of curvature is 2. AS per IS classification, this soil is classified as

(a) SP (b) GP

(c) SW (d) GW

Q.7 In a particular soil sample, laboratory analysis has yielded the following result:

1. Sand – 20%
2. Silt – 30%
3. Clay – 50%

Without using the textural chart, the correct textural classification of the soil would be

(a) loam (b) sandy clay

(c) silty loam (d) clay

Q.8 Sieve analysis on a dry soil sample of mass 1000 g showed that 980 g and 270 g of soil pass through 4.75 mm and 0.075 mm sieve, respectively. The liquid limit and plastic limits of the soil fraction passing through 425 m sieves are 40% and 18%, respectively. The soil may be classified as

(a) SC (b) MI

(c) CI (d) SM

Q.9 Consider the following statements:

1. Coarse-grained soil having fines (<75μ in size) between 5% and 12%, have a dual symbol according to IS code for soil classification
2. At liquid limit, all soils have the same shearing strength.
3. Lower the shrinkage limit, greater is the volume change in a soil with change in water content.

Which of these statements are correct?

(a) 1 and 2 (b) 1 and 3

(c) 2 and 3 (d) 1, 2 and 3

Answers

1. (c) 2. (c) 3. (b) 4. (c) 5. (c)

6. (c) 7. (d) 8. (a) 9. (d)

Hints and Explanations:

1. (c)

A	B
$PI = LL - PL$	$60 - 35 = 25$
$55 - 20 = 35$	

$$e = \frac{wG}{S} \qquad \frac{0.4 \times 2.65}{0.8} = 1.325$$

$$\frac{0.25 \times 2.7}{1} = 0.675$$

$$\gamma_d = \frac{G\gamma_w}{1+e} \qquad \frac{2.65 \times 1}{1+1.325} = 1.14 \text{ g/ml}$$

$$\frac{2.7 \times 1}{1+0.675} = 1.61 \text{ g/ml}$$

PI of *B* is more hence it has more clay particles.

3. (b)

GW is well graded gravel for which coefficient of uniformity $(C_u) > 4$.

SW is well graded sand for which coefficient of uniformity $(C_u) > 6$

ML is silt with low plasticity (< 35%)

CL is clay with low plasticity (< 35%). It also possess low compressibility.

5. (c)

Particle Size	% Retained	% Cumulative	% N = 100-cum. %
Gravel retained 4.75 mm	40%	40%	60%
Sand retained 0.075 mm	50%	90%	10%
Silt retained 0.002 mm	10%	100%	0%

$$\therefore \qquad D_{60} = 4.75 \text{ mm}$$

$$D_{10} = 0.075 \text{ mm}$$

$$\therefore \qquad C_u = \frac{D_{60}}{D_{10}} = \frac{4.75}{0.075} = 63.33 > 60$$

So correct answer is '*c*'.

6. (c)

Since more than 50% of particles are passing through 4.75 mm sieve while less than 50% are passing through 75μ sieve, the soil is sand.

$$C_u = 8 > 6$$

$$1 < C_c = 2 < 3$$

Therefore its well graded sand (SW)

7. (d)

As 50% of soil is clay. So it will be classified as clay.

9. (d)

Lower the shrinkage limit greater is the volume change.

For coarse grained soil with fines < 5% classification will be GP, GW, SP, and SW. for fines > 12% classification will be based on plasticity chart as GM, GC, SM and SC. For fines 5 – 12% dual classification like GP – GM; GP – GC etc., will be used.

At liquid limit the soils possess a certain shear strength which is the smallest value that can be measured in a standard procedure. From direct shear tests on different types of clays it is found that liquid limit corresponds to a shearing strength of about 2.7 kN/m².

$$\text{The shrinkage ratio SR} = \frac{(V_1 - V_2)/V_d}{W_1 - W_2} \times 100$$

$$\frac{(V_1 - V_2)}{W_1 - W_2} \times 100 = SRV_d = \frac{1}{\left(w_s + \frac{1}{G}\right)}$$

If shrinkage limit is less the volume change with change in water content will be more.

CHAPTER 04

Soil Structure and Clay Minerals

4.1 Introduction

The coarse grained soils generally contain the minerals quartz and feldspar. These minerals are strong and electrically inert. The behaviour of such soils does not depend upon the nature of mineral present; on the other hand, behaviour of fine grained soils depends on large extent on the nature and minerals present. The most significant properties of clay depends upon the type of minerals. The crystalline mineral whose surface activity is high are clay minerals. These clay minerals impart cohesion and plasticity. So the study of clay minerals is essential for understanding the behaviour of clayey solids.

4.2 Clay Minerals

The clay minerals are hydrous aluminium silicate with other metallic ions in a sheet like structure. Their particles are very small in size, very flaky in shape and thus have considerable surface area. These clay minerals are evolved mainly from the chemical weathering of certain rock minerals.

4.3 Structure of Clay Minerals

Clay minerals are composed of two basic structural units.

(i) Tetrahedral unit (ii) Octahedral unit

(i) **Tetrahedral Unit:** The tetrahedral unit comprises of a central silicon atom surrounded by four oxygen atoms positioned at the vertices of the tetrahedron.

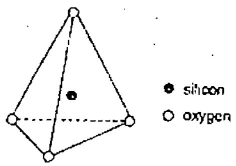


Fig. 4.1 Single tetrahedral unit

The tetrahedral units are combined with each other such that each oxygen atom at the base of tetrahedron lies in same plane and is being shared between two tetrahedron units. This combination of tetrahedral units is called silica sheet.

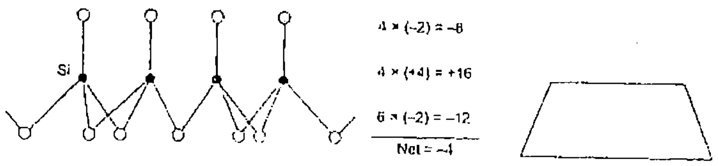


Fig. 4.2 Symbolic representation of silica sheet

In above silica sheet, each of the oxygen ion at the base is common to two adjacent units. The sharing of charges leaves three negative charges at the base per tetrahedral unit. This, alongwith the two negative charges at the apex, makes a total of five negative charges to four positive charges of silicon ion. Thus, there is a net charge of –1 per unit.

(ii) **Octahedral unit:** The octahedral unit comprises a central ion of either aluminium, magnesium or iron surrounded by six hydroxyl ions.

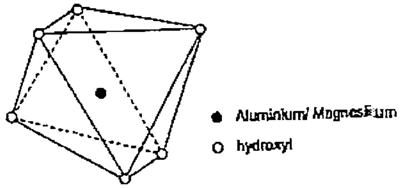


Fig. 4.3 Single octahedral unit

The octahedral sheet is a combination of octahedral units. If the atom at the centre is Aluminium, the resulting sheet is called Gibbsite sheet. If magnesium is the central atom, the sheet is called the Brucite sheet. If iron is the central atom, the sheet is called Ferrite sheet.

Each hydroxyl ion in Gibbsite sheet is being shared between 3 octahedral units. Therefore, net charge present over Gibbsite is –1.

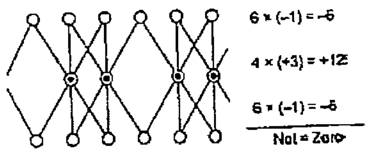


Fig. 4.4 Diffused double layer

4.4 Isomorphous Substitution

It is possible that one atom in a basic unit may be replaced by another atom. The process is known as isomorphous substitution. For example, one silicon ion in a tetrahedral unit substituted may lead to increase in negative charge on the particle which affect the physical properties of mineral.

4.5 Types of Clay Minerals

Clayey soils are made of three basic minerals

(i) Kaolinite mineral

(ii) Montmorillonite mineral

(iii) Illite mineral

(i) **Kaolinite Minerals:** Kaolinite minerals are made of an aluminium sheet (gibbsite sheet, Gi) combined with a silica sheet (Si). The structural units join together by hydrogen bond, which develops between the oxygen of Silica sheet and the hydroxyls of Alumina sheet. Since hydrogen bond is strongest bond, Kaolinite mineral is quite stable. Thus soils consisting kaolinite mineral do not show much change in volume due to moisture variation. The most common example of kaolinite mineral is china clay. The surface area of the Kaolinite particles per unit mass is about 15 m²/g. The surface area per unit mass is defined as specific surface.

(ii) **Montmorillonite Minerals:** Montmorillonite mineral is a three layer sheet mineral. Basic three layer sheet units are formed by keeping one silica sheet (Si) on the top and bottom of gibbsite sheet (Gi).

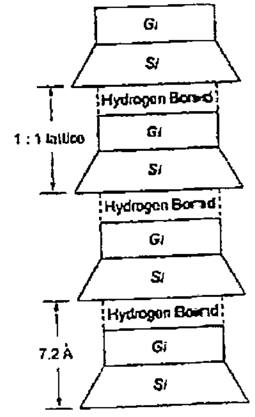


Fig. 4.5 Kaolinite Mineral

Isomorphous substitution of magnesium or iron for the aluminium in the gibbsite sheet is common. It attracts water to form a water layer between two connecting basic units. The bonding between basic units is by water forces and exchangeable ion linkage. The bonding of these sheets is very weak, and water may enter between the units. Thus mineral has significant affinity for water, and results in expansion and swelling. Bentonite and Black cotton soils contains montmorillonite mineral in large proportion.

Specific surface of montmorillonite is about 200-800 m²/g.

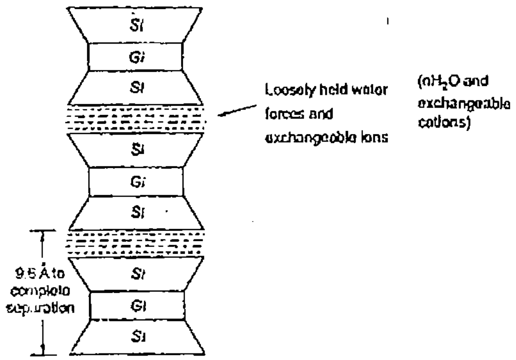


Fig. 4.6 Montmorillonite mineral

(iii) Illite Minerals

Illite is also a three-layer sheet mineral. It consists of basic three layer sheet unit similar to montmorillonite. There is also a substantial amount of isomorphous substitution of silicon by aluminium in silica sheet. Consequently, the mineral has a larger negative charge than that in montmorillonite. The basic units are bonded by non-exchangeable potassium (K⁺) ions, which is stronger than the bond in montmorillonite. Thus illite swells less than montmorillonite. However swelling is more than in kaolinite.

The specific surface is 50-100 m²/g.

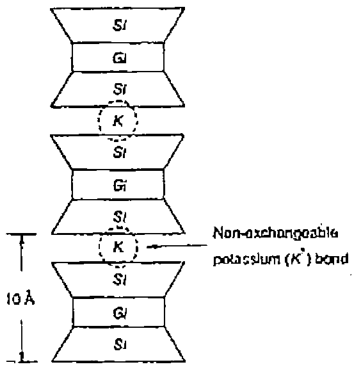


Fig. 4.7 Illite mineral

4.6 Clay Water Relationship

Clay soils are normally associated with water and their properties are greatly influenced by the presence of water. Whereas, there is no impact of structural water on granular soils except for reduction in void due to submergence. The surface of clay particles carry negative charges. The edges of a clay particle may have a positive or a negative charge. Because of net negative surface charges, the clay particles repel each other, but edge to surface attraction may occur. Generally clay particles attract cations (positive ions).

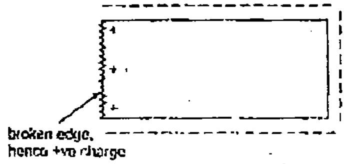


Fig. 4.8 Clay particle

Fig. 4.9 Water molecule

The water immediately surrounding a clay mineral is a dipolar molecule thus treated as a bar magnet. Further to balance the negative charge near the surface layer of clay molecule, cations from water molecule are attracted. This attraction decreases with distance from the surface. Hence there is no attraction on pore water.

The distance from the surface of the particle to the limit of attraction is termed as Diffuse double layer. The water contained between the diffuse layer and adjacent to the surface of clay is termed as Rigid layer.

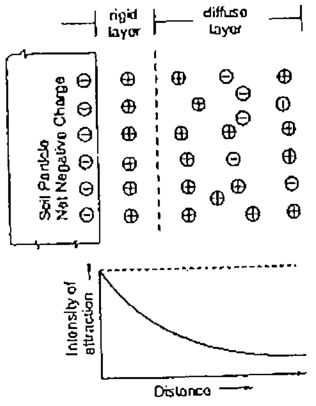


Fig. 4.10 Diffuse Double Layer

NOTE The surface of clay particles carry a negative charge. This results from any one of the combination of the following factors:
(i) isomorphous substitution
(ii) surface dissociation of OH⁻
(iii) absence of cation in the crystal lattice
(iv) adsorption of anion
(v) presence of organic matter

4.7 Clay Particle Interaction

The clay particle interact through the adsorbed water layers. Therefore, factors such as the nature of ions present, their concentration and size and other environmental conditions influence the soil structure found in natural clay deposits.

The clay particles in aqueous environment may mutually attract or repulse. If the total potential energy between the particle decreases as they approach each other, there is attraction between them in which case the particle 'Flocculate'. If the approaching particles increase the energy of the system, they move apart or 'Disperse'.

Particles generally flocculate essentially in the edge to face configuration.

Dispersion of particles will be mostly in the face to face configuration.

The tendency to flocculate increase with increases of one or more of the following:

(i) Concentration of the electrolyte (ii) Valency of the ion
(iii) Temperature

Tendency to Flocculate increase with decreases of one or more of the following:

(i) Dielectric constant of the medium (ii) Size of the ion
(iii) pH (iv) Anion adsorption

The dispersed clay with basic face to face arrangement of platelets has a lower void ratio than the flocculated clay, which has an appreciably large proportion of voids.

4.8 Soil Structure

Soil structure is a more generalized term, applicable to all types of soil. This includes:

(i) Particle gradation
(ii) Compactness

(iii) Interparticle forces and bonding agents
(iv) Geometric and skeletal arrangement of particles

There are various factors that affect the structure of coarse grained soils:

(i) Particle size distribution
(ii) Gravity

There are various factors that affect the structure of clay soils:

(i) Type of clay mineral
(ii) Surface forces

4.9 Types of Soil Structures

4.9.1 Single Grained Structure

In granular soils, the ratio of volume to the surface area is large, so that mass derived (i.e. gravity) forces are dominant and surface-derived electric forces are negligible. These are found in soils having size greater than 0.02 mm. Single-grained structures are formed when the soil grains settle out independently due to gravity force. Examples - gravel and coarse sand

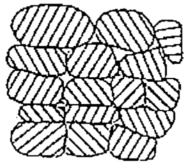


Fig. 4.11 (a) Single grained structure

4.9.2 Honeycomb Structures

This type of structures are found in soil having size between 0.02 mm to 0.002 mm. Gravity and surface electric (adhesive) forces, both are responsible for their formation. These structures enclose large volume of voids. When structure is unbroken, these soils have ability to bear large loads, but once the structure is broken, load carrying capacity is lost and show large deformation. Examples - fine sand and silts.

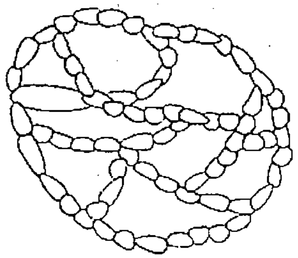


Fig. 4.11 (b) Honey comb structure

4.9.3 Flocculated and Dispersed Structures

These are found in the soils having size less than 0.002 mm. The structure of a fine-grained cohesive soil can be described fully with the understanding of inter particle forces and the geometrical arrangement of particles.

The fine particles are of flaky or plate like shaped. They have large surface area and therefore surface electric force become dominant. The clay particles have a negative charge on the surface and positive charge on edges. Particles joined edge-to-edge or edge-to-surface results in a flocculated structure. This soils structure have high volume of voids.

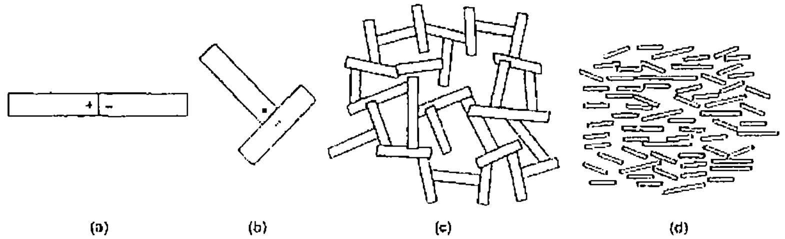


Fig. 4.12 (a) Edge-to-edge *(b)* Edge-to-surface *(c)* Flocculated structure *(d)* Dispersed structures

Dispersed structures develops in clays that have been remoulded. When flocculated structural soils are remoulded by nature or man, converts its edge-to-edge or edge-to-surface orientation into surface-to-surface orientation, this results in a dispersed structure.

4.9.4 Structure of a Composite Soil

Composite soils are mixture of coarse grain and fine soils.

(a) Coarse Grained Skeleton: Coarse grained skeleton is a composite structure which is formed when soil contain both coarse grain and fine grain particles.

Coarse grain particles form a frame work or skeleton. The space between these skeleton is filled by fine grain particles called binders. These types of soils are less compressible.

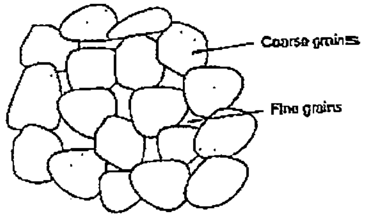


Fig. 4.13 Coarse grain skeleton

(b) Clay-matrix Structure: Clay-matrix structure is also a composite structure which is formed when soil contain both fine grain and coarse grain soils. However, in this case the amount of fine particles is very large as compared to coarse particles. The clay forms a matrix in which coarse grain appear floating without touching each other. Such soils are relatively more compressible.

Summary

- Interparticle forces acting between particles depends upon the surface area, its character and environment.
- In the formation of coarse soils gravity forces are dominant where as surface electric charge are dominant in formation of clays.
- Clay minerals are made of Silica and Gibbsite sheets.
- The three important clay minerals are
  (i) Kaolinite mineral
  (ii) Montmorillonite mineral
  (iii) Illite mineral
- Flocculated and dispersed structure are the two basic structures of clay.
- Single grain and honey comb structure are the two structure of sand.

Objective Brain Teasers

Q.1 The description of 'sandy silty clay' signifies that
(a) the soil contains unequal proportions of the three constituents, in the order sand > silt > clay
(b) the soil contains equal proportions of sand, silt and clay
(c) the soil contains unequal proportions of sand, silt and clay
(d) the soil contains unequal proportions of the three constituents such that clay > silt > sand

Q.2 The correct sequence of plasticity of minerals in soil in an increasing order is
(a) silica, kaolinite, illite, montmorillonite
(b) kaolinite, silica, illite, montmorillonite
(c) silica, kaolinite, montmorillonite, illite
(d) kaolinite, silica, montmorillonite, illite

Q.3 Among the clay minerals, the one having the maximum swelling tendency is
(a) Kaolinite (b) Illite
(c) Montmorillonite (d) Halloysite

Q.4 Among the following types of water, which one is chemically combined in the crystal structure of the soil mineral and can be removed only by breaking the crystal structure?
(a) Capillary water
(b) Adsorbed water
(c) Hygroscopic water
(d) Structural water

Q.5 Honey-combed structure is found in
(a) Gravels
(b) Coarse sands
(c) Fine sands and silts
(d) Clay

Q.6 Which one of the following is the most active clay material?
(a) Na-illite
(b) Na-kaolinite
(c) Na-montmorillonite
(d) Ca-montmorillonite

Q.7 The plasticity characteristics of clay are due to
(a) Adsorbed water
(b) Free water
(c) Capillary water
(d) None of the above

Answers

1. (d) 2. (a) 3. (c) 4. (d) 5. (c)
6. (c) 7. (a)

Hints and Explanations:

2. (a)
Silica has least plasticity while Montmorillonite has highest plasticity.

3. (c)
Montmorillonite has highest swelling characteristics and kaolinite has lowest swelling characteristics

■■■■

CHAPTER 05

Soil Compaction

5.1 Introduction

Construction of many structures requires "stabilisation" of the soil mass i.e. an artificial improvement of its engineering properties. There are various methods of soil stabilisation, the most common being the mechanical stabilisation and the simplest technique of mechanical stabilisation is compaction.

Compaction is a process by which the soil particles are artificially rearranged and packed together into a closer state of contact by mechanical means in order to reduce the volume of air voids of the soil and thus increase its dry density.

A soil mass can be compacted by either a dynamic process or a static one. In the dynamic method the soil mass is compacted by repeated applications of a dead load, while in static method, compaction is done by a steady increase of static load.

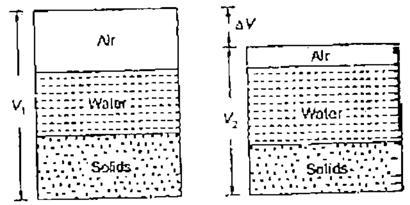
Do you know? The dynamic method gives better result in coarse grained soils and static method is suitable for less permeable fine grained soils.

5.2 Principles of Compaction

Usually when soil is excavated from one place, transported and dumped in another place it is loosened up to a great extent. In an uncompacted fill, the void ratio is relatively high and the corresponding density is low. Consequently, the shearing strength of uncompacted soil is low while its permeability is high. A structure on such a fill may crack extensively due to non uniformity of settlement. Therefore, it is necessary to artificially compact the fill.

Soil compaction is the process where soil particles are forced to pack more closely by reducing air voids. This can be achieved by applying some mechanical energy (static or dynamic) on soil fill.

The degree of compaction of a soil is measured in terms of dry unit weight i.e. the amount of soil solids that can be packed in a unit volume of the soil.



(a) Before compaction (b) After compaction

Fig. 5.1

Remember: Compaction is somewhat different from consolidation. In consolidation volume reduction takes place due to expulsion of pore water from saturated voids.

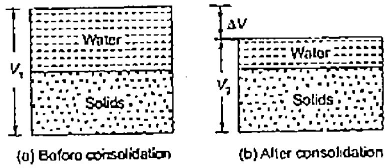
5.3 Difference between Compaction and Consolidation

5.3.1 Compaction

- Almost an instantaneous phenomenon.
- Soil is always unsaturated.
- Densification is due to a reduction in the volume of air voids at a given water content.
- Specified compaction techniques are used in this process.

5.3.2 Consolidation

- It is a time dependent phenomenon.
- Soil is completely saturated.
- Volume reduction is due to expulsion of pore water from voids.
- Consolidation occurs on account of a load placed on the soil.



(a) Before consolidation (b) After consolidation

Fig. 5.2

5.4 Advantages of Compaction

Proper compaction of a soil mass lead to

- Increase in shear strength of soil.
- Improved stability and bearing capacity of soil.
- Reduction in compressibility and permeability of soil.
- Increase in the load carrying capacity of soil subgrade.
- Prevention of determental-settlements and undesirable volume changes through swelling and shrinkage.

Do You Know When a heavily compacted soil mass (near to maximum dry unit weight) is sheared, it tends to expand and gets loosened. Therefore, heavily compacted soils tend to show decrease in strength when sheared.

5.5 Laboratory Compaction

Laboratory compaction tests are designed to estimate the dry density of soils. These tests are based on any one of the following methods or type of compaction: dynamic or impact, kneading, static and vibration. The tests are performed by compacting a wet soil sample in a mould in a specified number of layers. Each layer is compacted at specified number of blows. After compaction of final layer, the bulk density of soil and corresponding moisture content are determined. Test are repeated on fresh samples with changed moisture content. With known values of bulk density and moisture content, the dry density may be calculated as given below.

$$\text{Dry density, } \rho_d = \frac{\rho_t}{1+3}$$

Now a graph of dry density Vs moisture content is plotted and the maximum dry density and optimum moisture content are read from the graph.

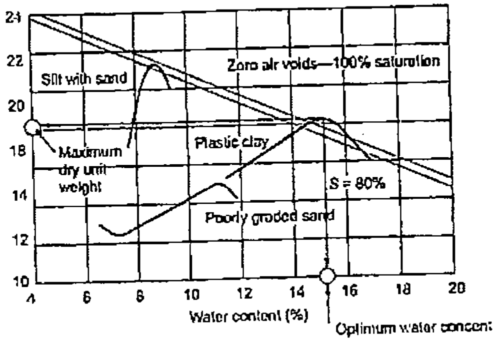


Fig. 5.3

5.5.1 Standard Proctor Test

- Standard volume mould (944 cc or 1/30 cubic feet).
- Filled up with soil in three layers.
- Each layer is compacted by 25 blows of standard hammer (weight 2.495 kg or 5.5 lbs) falling through 304.8 mm (12 inch).
- Dry weight is calculated by knowing the wet weight of compacted soil and its water content.

  Dry unit weight,

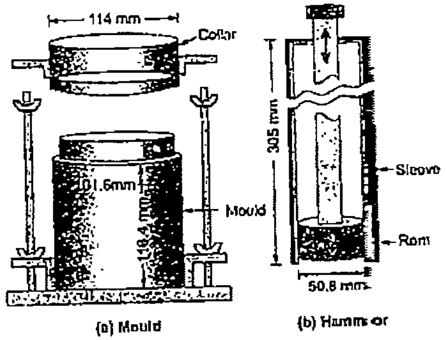
$$\gamma_d = \frac{\gamma}{1+w}$$

where, γ = bulk unit weight

$$= \frac{\text{weight of compacted soil}}{\text{volume of the mould}}$$

w = water content

- Test is repeated at different water contents.
- Compaction curve is plotted between moisture content and dry unit weight.
- The peak of compaction curve corresponding to the maximum dry unit weight is referred as $\gamma_{d(max)}$.
- The moisture content corresponding to $\gamma_{d(max)}$ is known as optimum moisture content (OMC) at a given compactive effort.



(a) Mould (b) Hammer

Fig. 5.4 Standard Proctor

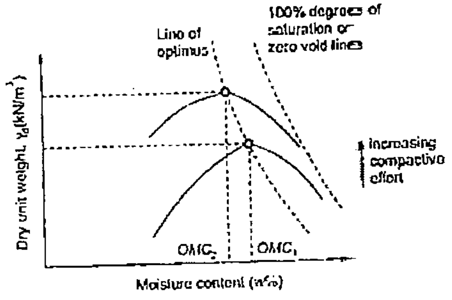


Fig. 5.5 Compaction Curve

NOTE

- Typical values of maximum dry unit weights range from 16 to 20 kN/m³ with the widest range being 13 to 24 kN/m³.
- Typical optimum moisture content values ranges from 10 to 20% with a maximum range of 5 to 30%.
- Maximum dry unit weight so obtained is only for a given amount of compaction effort and method of compaction. It is also not necessary that the maximum dry unit weight can be obtained in the field.

The specifications for compaction of fills in the field are usually based on maximum dry unit weight but sometimes on both the maximum dry unit weight and the OMC.

5.5.2 Modified Proctor Test

- Developed during World War II, to simulate the compaction required for air fields to support heavier aircrafts.
- Standard volume mould (944 cc or 1/30 cubic feet).
- Filled up with soil in five layers.
- This test employs heavier hammer (4.54 kg or 10 lbs).
- Height of fall is 457.2 mm (18 inches) and each layer is tamped 25 times into a standard proctor mould.

5.5.3 Indian Standard Equivalent of Standard Proctor Test (Light Compaction Test)

- Volume of mould is 1000 cc.
- Weight of hammer is 2.6 kg and drop is 310 mm.
- Mould is filled in three layers and each layer tamped by 25 blows.

5.5.4 Indian Standard Equivalent of Modified Proctor Test (Heavy Compaction Test)

- Volume of mould is 1000 cc.
- Weight of hammer is 4.9 kg and height of drop is 450 mm.
- Soil is compacted in 5 layers and each layer tamped by 25 blows.

Do you know? Under heavier compaction the moisture-density curve is shifted upward and simultaneously towards the left, resulting in a lower OMC but a greater $\gamma_{d(max)}$.

5.6 Comparison of Standard and Modified Proctor Test

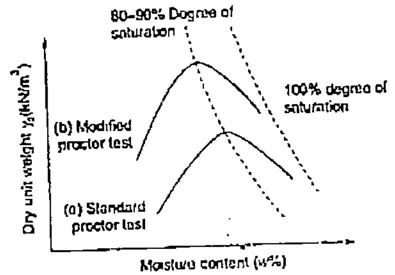


Fig. 5.6

Remember

- Compactive energy applied in proctor test per unit volume is given by

$$E = \frac{nNWh}{V}$$

where,

n = Number of layer
N = Number of blows per layer
W = Weight of hammer
h = Height of free fall of hammer
V = Volume of mould

- Compactive energy per unit volume in Standard Proctor Test

$$E = \frac{3 \times 25 \times 2.495 \times 0.3048 \times 9.81}{944 \times 10^{-6}} = 592.71 \text{ kJ/m}^3$$

- Compactive energy per unit volume in Modified Proctor Test

$$E = \frac{5 \times 25 \times 4.54 \times 0.4572 \times 9.81}{944 \times 10^{-6}} = 2696.30 \text{ kJ/m}^3$$

- Compactive energy per unit volume in Light Compaction Test

$$E = \frac{3 \times 25 \times 2.6 \times 0.310 \times 9.81}{1000 \times 10^{-6}} = 593.01 \text{ kJ/m}^3$$

- Compactive energy per unit volume in Heavy Compaction Test

$$E = \frac{5 \times 25 \times 4.9 \times 0.45 \times 9.81}{1000 \times 10^{6}} = 2703.88 \text{ kJ/m}^3$$

Example 5.1 The compaction of a floor area is carried out in 250 mm thick layers. The hammer used for compaction has the foot area of 0.05 m². The energy developed per drop of the rammer is 50 kg-m. Assuming 40% more energy in each pass over the compacted area due to overlap, calculate the number of passes required to develop compactive energy equivalent to IS heavy compaction for each layer.

Solution:

Compactive energy as per IS heavy compaction test

$$= \frac{4.9(\text{kgf}) \times 0.45(\text{m}) \times 5(\text{layers}) \times 25(\text{blows/layer})}{10^3 \times 10^{-6}(\text{m}^3)} = 27562.5 \text{ kgfm/m}^3$$

Compactive energy per drop provided by the rammer per m³ of the soil

$$= \frac{50}{0.05 \times 250 \times 10^{-3}} = 4000 \text{ kgfm/m}^3$$

However, in each pass over a layer, the energy supplied will be 1.4 times this value on account of overlap of hammer foot print. Let n be the number of passes required to develop compactive energy equivalent to IS heavy compaction.

$$\therefore \quad n \times 1.4 \times 4000 = 27562.5$$

$$n = 4.92 \text{ say } 5$$

5.7 Zero Air Void Line

At constant moisture content, the dry unit weight reaches its theoretical maximum when all the air is expelled from the void spaces, i.e. when degree of saturation is 100%. Therefore the zero air void line is a line joining points having dry unit weights corresponding to 100% saturation at different moisture contents. Therefore, it is also called the saturation line.

Zero air void lines can be defined as "The lines showing the dry density as a function of water content for soil containing no air voids."

We can derive its equation as follows:

$$\gamma_d = \frac{G_s\gamma_w}{1+e} \quad \text{...(i)}$$

For any degree of saturation,

$$e = \frac{wG_s}{S}$$

Therefore we can write the expression for dry unit weight corresponding to any degree of saturation S as

$$\gamma_{d,s} = \frac{G_s\gamma_w}{1+\frac{wG_s}{S}} = \frac{\gamma_w}{\left(\frac{1}{G_s}\right)+\left(\frac{w}{S}\right)} \quad \text{...(ii)}$$

where, $\gamma_{d,s}$ = dry unit weight at degree of saturation
γ_w = unit weight of water, e = void ratio
w = water content, G_s = sp. gravity of solids

For zero air voids, degree of saturation becomes 100%. Therefore equation (ii) becomes

$$\gamma_{d_0} = \frac{\gamma_w}{\left(\frac{1}{G_s}\right)+w} \quad \text{...(iii)}$$

where, γ_{d_0} = dry unit weight at zero air void

Remember: Zero air voids line or saturation line is always a steadily decreasing line.

5.8 Constant Percentage Air Void Lines

These are lines which shows the water content, dry density relation for the compacted soil containing a constant percentage air void is known as an air voids line.

By the definition of percentage air voids, we have

$$\frac{n_a}{100} = \frac{V_a}{V_v} = \frac{V_v - V_w}{V_v} = 1 - S$$

or

$$S = 1 - \frac{n_a}{100} \quad \text{...(iv)}$$

Substituting value of S into equation (ii), we have

$$\gamma_{d,a} = \frac{G_s\gamma_w\left(1-\frac{n_a}{100}\right)}{\left(1-\frac{n_a}{100}\right)+wG_s}$$

where, $\gamma_{d,a}$ = dry unit weight at constant percentage air voids

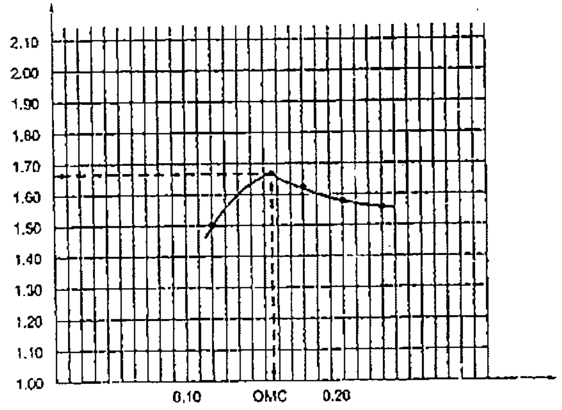
Example 5.2 : The following results were obtained from a IS Light Compaction Test on sample of soil

Water content (%)	0.12	0.15	0.18	0.21	0.24
Mass of Wet Soil (kg)	1.65	1.95	1.93	1.90	1.86

Plot the compaction curve. Hence obtain the maximum dry density and the optimum moisture content. Also calculate the void ratio the degree of saturation and theoretical maximum dry density (G = 2.68).

Solution:

Water Content (w)	0.12	0.15	0.18	0.21	0.24
Mass of wet soil M (kg)	1.65	1.95	1.93	1.90	1.86
Bulk density $\rho = \frac{M}{V} = \frac{M}{1.0}$ kg/m^3	1.65	1.95	1.93	1.90	1.86
Dry density $\rho_d = \frac{\rho}{1+w}$	1.47	1.69	1.67	1.57	1.5
Void ratio $e = \frac{G\rho_w}{\rho_d} - 1$	0.62	0.58	0.60	0.70	0.79
Degree of saturation $S = \frac{wG}{e}$	0.39	0.69	0.80	0.604	0.81
Theoretical max dry density $\rho_{d(max)} = \frac{G\rho_w}{1+wG}$	2.03	1.91	1.81	1.71	1.63



From the compaction curve, $\rho_{d(max)}$ = 1.66 g/cc

Optimum moisture content, OMC = 0.163 or 16.3%

Example 5.3 The results of a standard compaction test are shown in the table below. Determine the maximum dry unit weight and optimum water content.

Water Content (%)	6.2	8.1	9.8	11.5	12.3	13.2
Bulk unit weight (kN/m^3)	16.9	18.7	19.5	20.5	20.4	20.1

(a) What is the dry unit weight and water content at 95% standard compaction?

(b) Determine the degree of saturation at the maximum dry density.

(c) Plot the zero air voids line.

Compute γ_d and then plot the results of γ_d versus w(%). Then extract the required information.

Solution:

(a)

Water Content (%)	6.2	8.1	9.8	11.5	12.3	13.2
Bulk unit weight (kN/m^3)	16.9	18.7	19.5	20.5	20.4	20.1
Dry unit weight (kN/m^3) $\gamma_d = \frac{\gamma}{1+w}$	15.9	17.3	17.8	18.4	18.2	17.8

$$\gamma_{d(max)} = 18.4 \text{ kN/m}^3$$

and optimum moisture content,

$$w_{opt} = 11.5\%$$

From Graph below,

$\therefore$ At 95% compaction, $\gamma_d = 18.4 \times 0.95 = 17.5 \text{ kN/m}^3$

Water content, $w = 9.2\%$

(b) Degree of saturation at maximum dry unit weight

We know, $$\gamma_d = \frac{G_s\gamma_w}{\left(1+\frac{wG_s}{S}\right)}$$

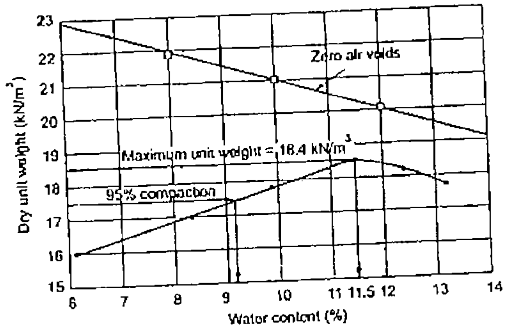
$$S = \frac{wG_s\gamma_{d(max)}/\gamma_w}{G_s-\gamma_{d(max)}/\gamma_w}$$

$$= \frac{0.115\times2.7\times(18.4/9.8)}{2.7-(18.4/9.8)}$$

$$= 0.71 \text{ or } 71\%$$

(c) Zero air void lines

Water Content (%)	6	8	10	12	14
Dry unit weight (kN/m^3) $\gamma_d = \frac{G_s\gamma_w}{\left(1+\frac{wG_s}{S}\right)}$ at $S = 1$	22.8	21.8	20.8	20.0	19.2



Example 5.4 The maximum dry density of a sample by the standard compaction test is 1.76 g/cc at an optimum moisture content of 14.5%. Find the air voids and the degree of saturation (G = 2.68).

What would be the corresponding value of dry density on the zero air void line at optimum moisture content?

Solution:

(i) We know,

$$\rho_d = \frac{\left(1-\frac{n_a}{100}\right)G\rho_w}{1+wG}$$

$$1.76 = \frac{\left(1-\frac{n_a}{100}\right)\times2.68\times1}{1+0.145\times2.68}$$

$$\therefore \quad n_a = 0.088 \text{ or } 8.80\%$$

(ii)

$$\because \quad \rho_d = \frac{G\rho_w}{1+e} = \frac{G\rho_w}{1+\frac{wG}{S}}$$

$$1.76 = \frac{2.68\times1}{1+\left(\frac{0.145\times2.68}{S}\right)}$$

$$1.76+\frac{0.683}{S} = 2.68$$

$$S = 0.7423 \text{ or } 74.23\%$$

(iii) At zero air void line, S = 100%

$$\therefore \quad \rho_{d(theo,max)} = \frac{G\rho_w}{1+\frac{wG}{S}} = \frac{G\rho_w}{1+wG} = \frac{2.68\times1}{1+0.145\times2.68} = 1.93 \text{ g/cc}$$

5.9 Factors Affecting Compaction

The major factors which affect compaction are:

(i) water content	(ii) compactive effort
(iii) type of soil	(iv) method of compaction

(i) **Water Content:** At lower water contents, the soil particles offer more resistance to compaction and soil behaves like a stiff material. Increasing the moisture content helps the particles to move closer because of the lubrication effect. On further increasing the moisture content beyond a certain limit, the water starts to replace the soil particles. Thus the dry unit weight continuous to increase till the Optimum Moisture Content (OMC) is reached and with further increase in moisture content beyond OMC, unit weight starts decreasing.

Do You Know?

Using, $\gamma_d = \frac{G\gamma_w}{1+e} = \frac{G\gamma_w}{1+\frac{wG}{S}}$

we conclude $\gamma_d \propto S$ i.e. degree of saturation.

Therefore for a given water content, the theoretical maximum value of dry unit weight for a compacted soil is obtained corresponding to the situation when no air voids are left i.e. when degree of saturation becomes 100%.

(ii) **Compactive Effort:** For a given type of compaction, an increase in the amount of compaction will initially result in closer packing of the soil particles and maximum dry unit weight increase while the optimum moisture content at which it is attained decreases.

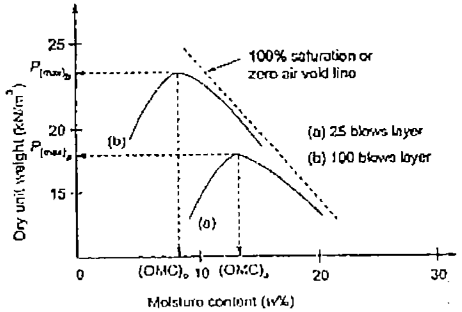


Fig. 5.7 Effect of compactive effort

(iii) Type of Soil

- Coarse grained soils, well graded can be compacted to high dry unit weight, especially if they contains some fines.
- However, if quantity of fines is excessive the maximum dry unit weight decreases.
- Poorly graded or uniform sands lead to lowest dry unit weight value.
- In clayey soils, maximum dry unit weight tends to decrease as plasticity increases.
- Cohesive soils have generally high value of OMC.

- Heavy clays with high plasticity have very low value of maximum dry unit weight and very high value of OMC.

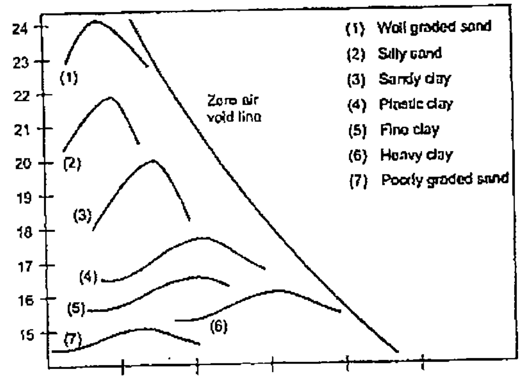


Fig. 5.8 Typical proctor compaction curve for different soils

(iv) **Method of Compaction:** Since the field compaction is essentially a kneading or rolling type compaction whereas the laboratory tests are dynamic-impact type compaction, therefore, laboratory compaction tests have more value of maximum dry unit weight.

5.10 Compaction Behaviour of Sand

The moisture-dry density relationship, as obtained from a laboratory test on a cohesionless sand is shown in figure below:

- Initially there is decrease in dry unit weight with the increase in water content. This is due to bulking of sand i.e. capillary tension in pore water prevents soil particles coming closer. *The maximum bulking occurs at 4 - 5% water content.*
- The maximum dry unit weight occurs when soil is either completely saturated or dry.
- When water content is increased further, there is fall in dry unit weight again.

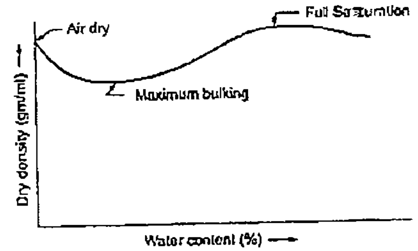


Fig. 5.9 Compaction Behaviour of Sand

Remember: Coarse grained soils do not absorb water, but fine grained soils do and hence the Lambe's double layer theory is not applicable to them.

5.11 Effect of Compaction on Properties of Soils

5.11.1 Soil Structure

- The water content at which the soils is compacted plays an important role in the engineering properties of the soils.

- At low water content, attractive forces between the particles are stronger than repulsive forces. Hence, soils compacted at a water content less than the optimum water content generally have a flocculated structure.
- Increasing the water content, increases the repulsive forces. Hence, soils compacted at water content more than the optimum water content usually have a dispersed structure.
- If the compactive effort is increased there is a corresponding increase in the orientation of the particles and higher dry densities are obtained, as shown by the upper curve.

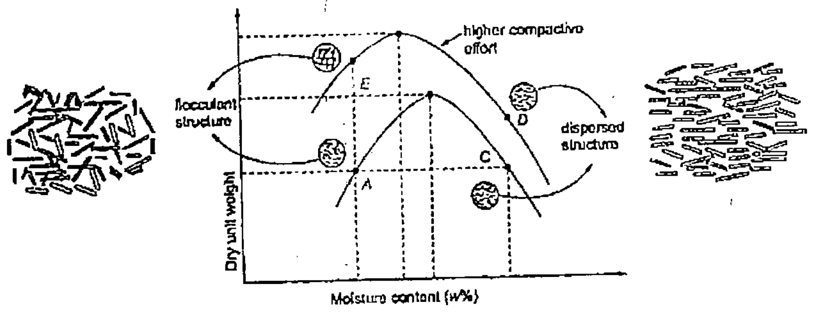


Fig.5.10

5.11.2 Permeability

- The permeability of a soil depends upon the size of voids. Due to increase in water content for a given compactive effort there is an improved orientation of the particles and a corresponding reduction in the size of voids which cause a decrease in permeability.
- For a given compactive effort, the permeability decreases sharply with increase in water content on the dry side of optimum. The minimum permeability occurs at or slightly above the OMC. Beyond OMC, the permeability may show a slight increase, but always remain much smaller than on the dry side of optimum.
- If the compactive effort is increased, the permeability of the soil decrease due to increased dry density and better orientation of particle.

5.11.3 Compressibility

- At relatively low stress levels, a soil compacted wet side (dispersed soil) of optimum is more compressible than the soil compacted dry side (flocculated soil) of optimum. Therefore, for construction of embankment, the soil is compacted on the dry side of optimum.
- At high stress levels, the compressibility increase due to breakdown of the structure, now the flocculated structures with larger void volume, can undergo a large volume decrease.

5.11.4 Swelling

- A soil on the dry side of optimum has a higher water deficiency and a more random particle arrangement. It can therefore, imbibe more water than a soil on the wet of optimum, therefore more swelling occur on dry side of optimum.

5.11.5 Shrinkage

- Soils compacted on the wet of optimum, have nearly parallel orientation of particles which allows the particles to pack more efficiently as compared to the randomly oriented particles on the dry side of optimum.
- Therefore, soils compacted on the wet of optimum tend to exhibit more shrinkage upon drying than those compacted on the dry of optimum.

5.11.6 Shear Strength

- The shear strength of the compacted soils depends upon the soil type, the moulded water content, drainage conditions, the method of compaction etc.
- In general, at a given water content, the shear strength of the soil increases with an increase in the compactive effort till a critical degree of saturation is reached. With further increase in the compactive effort, the shear strength decreases.

5.12 Field Compaction and Equipment

In the construction of highway embankments and earth dams, soil is first dumped in the form of loose fills, and then compacted to improve the density and the strength characteristics. In the field, soil is compacted by applying energy through mechanical equipment. The energy is transmitted to the soil by applying pressure in any one of the following three ways

(i) Static pressure (using rollers)
(ii) Impact (using rammers)
(iii) Vibration (using vibrator)

The choice of equipment will depend on the type of soil and economic consideration. The main compaction equipment along with suitability of soil and nature of project are summarized below:

S.No.	Type of Equipment	Suitability for Soil type	Nature of Project
1	Rammer or Tampers	All soils	In confined areas such as fills behind retaining walls, basement walls, trench fills etc.
2	Smooth wheeled rollers	Crushed rocks, gravels, sands	Road construction
3	Pneumatic tyred rollers	Sand, gravels, silts, clayey soils	Base, sub-base and embankment compaction for highways, air field and earth dams
4	Sheep foot rollers	Clayey soils	Core of earth dams
5	Vibratory rollers	Sands	Embankments for oil storage tanks etc.

5.13 Evaluation of Compaction

The method adopted to assess the degree of compaction obtained in the field is the relative compaction. It is the ratio of the dry unit weight attained in the field to specified standard unit weight expressed as a percentage. The specified standard unit weight may be proctor unit weight, Indian standard light or heavy unit weight.

$$\text{Relative compaction} = \frac{\gamma_{d(field)}}{\gamma_{d(max)}} \times 100$$

5.14 Compaction Quality Control

The compacted dry unit weights attained in the field should be checked occasionally for quality control. Three methods are generally used to check the in-situ density.

(i) Sand cone method
(ii) Rubber-balloon method
(iii) Nuclear density meters

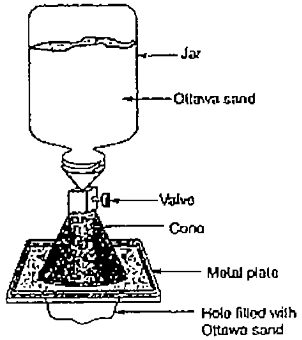


Fig. 5.11 Sand Cone Apparastus

(I) **Sand Cone Method:** A sand cone apparatus is shown in figure.

Procedure:

- Fill the jar with a standard sand of known density.
- Determine the weight of the sand cone apparatus with the jar filled with sand (say W_1).
- Determine the weight of sand to fill the cone (say W_2).
- Excavate the small hole in the soil and determine the weight of the excavated soil (say W_3).
- Fill the hole with the standard sand by inverting the sand cone apparatus over the hole and opening the valve.
- Determino the weight of the sand cone apparatus with the remaining sand in the jar (say W_4).
- Calculate the unit weight of the soil as follows:
  Weight of sand to fill hole,
  $$W_s = W_1 - (W_2 + W_4)$$
  Volume of hole, $$V = \frac{W_s}{\gamma_{d(sand)}}$$
  Weight of dry soil, $$W_d = \frac{W_3}{1+w}$$
  Dry unit weight, $$\gamma_d = \frac{W_d}{V}$$

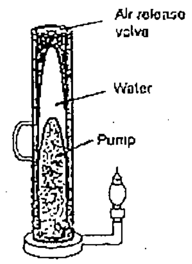


Fig. 5.12 Balloon Test Method

(ii) **Rubber-balloon Method:** Figure shows balloon test apparatus

Procedure:

- Fill the cylinder with water and record its volume, V_1.
- Excavate a small hole in the soil and determine the weight of the excavated soil (W).
- Determine the water content of the excavated soil (w).
- Use the pump to invert the balloon to fill the hole.
- Record the volume of water remaining in the cylinder, V_2.
- Calculate the unit weight of soil as follows:

$$\gamma = \frac{W}{V_1 - V_2}; \quad \gamma_d = \frac{\gamma}{1+w}$$

(iii) Nuclear Density Meter

- Nuclear density meter are used for direct determination of compacted unit weights and water content. Therefore, nuclear field density tests are much faster than conventional tests.
- Nuclear density meter works on the principle of scattering. Soil particles cause radiation to scatter to a detector tube and the amount of scatter is counted.
- The scatter count rate is inversely proportional to the unit weight of soil.
- If water is present in the soil, the hydrogen in water scatters the neutrons and the amount of scatter is proportional to the water content.

$$\therefore \quad \gamma_d = \frac{\gamma}{1+w}$$

5.15 Settlement During Compaction

Let e_0 be the initial void ratio before compaction has been started. After compaction, the void ratio of soil be e_f.

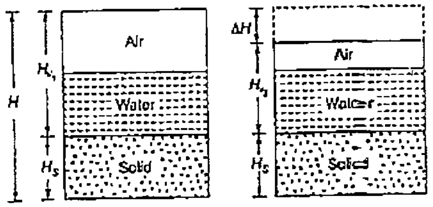


Fig. 5.13 Settling compaction

We know,

Void ratio, $$e = \frac{V_v}{V_s} = \frac{H_v}{H_s} \quad (\because V \propto H)$$

$$\therefore \quad e_0 = \frac{H_{V_1}}{H_s}$$

$$\Rightarrow \quad H_{V_1} = e_0 H_s$$

Similarly, $$H_{V_2} = e_f H_s$$

$\therefore$ change in the thickness of soil layer

$$\Delta H = H_{V_1} - H_{V_2} = (e_0 - e_f)H_s$$

$$\therefore \quad \frac{\Delta H}{H} = \frac{(e_0 - e_f)H_s}{H_s + e_0 H_s} = \frac{e_0 - e_f}{1+e_0}$$

$$\Delta H = \left(\frac{e_0 - e_f}{1+e_0}\right)H$$

Example 5.5 The unit weight of a compacted sand backfill was determined by field measurement to be 1738 kg/m³. The water content and void ratio of the laboratory compacted soil was 10.2% and 60.7% respectively. What was the degree of compaction achieved in field? Assume water content remain constant (G = 2.7).

Solution:

Given, $$\gamma_{(field)} = 1738 \text{ kg/m}^3$$
$$w = 10.2\%$$

For field, $$\gamma_d = \frac{\gamma_{(field)}}{1+w} = \frac{1738}{1+0.102} = 1577.13 \text{ kg/m}^3$$

For laboratory sample, $$w = 10.2\%$$
$$e = 0.607$$

We know,

$$\gamma_{(lab)} = \left(\frac{G+Se}{1+e}\right)\gamma_w = \frac{G(1+w)}{1+e}\gamma_w$$

$$= \frac{2.7(1+0.102)}{1+0.607} \times 1000 = 1851.5 \text{ kg/m}^3$$

$$\therefore \quad \gamma_{d(lab)} = \frac{\gamma_{(lab)}}{1+w} = \frac{1851.5}{1+0.102} = 1680.12 \text{ kg/m}^3$$

$$\therefore \quad \text{Degree of compaction} = \frac{\gamma_{d(field)}}{\gamma_{d(lab)}} \times 100 = \frac{1577.13}{1680.12} \times 100 = 93.87\%$$

Illustrative Examples

Example 5.6 Soil has been compacted in an embankment at a bulk density of 2.15 g/cc and the water content of 12%. The specific gravity of soil solids is 2.7. The water table is well below the foundation level. Estimate

(i) void ratio
(ii) the dry density
(iii) degree of saturation
(iv) air content of the compacted soil

Solution:

Given, $\rho = 2.15$ g/cc, $w = 12\%$ or 0.12, $G = 2.7$

(i) Using,

$$\rho = \frac{G(1+w)}{1+e}\rho_w$$

$$2.15 = \frac{2.7(1+0.12)}{1+e} \times 1$$

$$1+e = \frac{2.7(1+0.12)}{2.15} = 1.406$$

$$\therefore \quad e = 0.406$$

(ii)

$$\rho_d = \frac{\rho}{1+w} = \frac{2.15}{1+0.12} = 1.92 \text{ g/cc}$$

(iii) Using,

$$Se = wG$$

$$S = \frac{wG}{e} = \frac{0.12 \times 2.7}{0.406} = 0.708 \text{ or } 79.80\%$$

(iv)

$$\text{Air content} = \frac{V_a}{V_v} = \frac{V_v - V_w}{V_v} = 1 - S$$

$$= 1 - 0.798$$

$$= 0.202 \text{ or } 20.2\%$$

Example 5.7 During a compaction test, a soil attains the maximum dry unit weight of 18.6 kN/m³ at a water content of 15 percent. The specific gravity of soil is 2.7. Determine the degree of saturation, air content at the maximum dry unit weight. What would be the maximum theoretical dry density corresponding to zero air voids at optimum water content?

Solution:

Given, $\gamma_{d(max)} = 18.6 \text{ kN/m}^3$

OMC, $w = 15\%$ or 0.15

using,

$$\gamma_d = \frac{G\gamma_w}{1+\frac{wG}{S}}$$

$$18.6 = \frac{2.7 \times 9.81}{1+\frac{0.15 \times 2.7}{S}}$$

$$1+\frac{0.405}{S} = 1.424$$

$$S = 0.955 \text{ or } 95.5\%$$

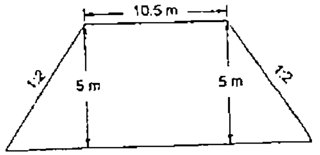
$$\text{Air content} = \frac{V_a}{V_v} = 1 - S$$

$$= 1 - 0.955 = 0.045 \text{ or } 4.5\%$$

At OMC of 15%, the theoretical maximum dry density will occur when air voids reduce to zero i.e. soil is completely saturated ($S = 1$).

$$\gamma_{d(max)} = \frac{G \cdot \gamma_w}{1+\frac{w \cdot G}{S}} = \frac{2.7 \times 9.81}{1+\frac{0.15 \times 2.7}{1}} = 18.85 \text{ kN/m}^3$$

Example 5.8 As per the compaction specification, a highway fill has to be compacted to 90% of Indian standard light compaction test dry density. A borrow pit available near the project site has a dry density of 1.78 g/cc at 100% compaction and a void ratio of 0.63. Compute the volume of borrow material needed to construct a highway fill of height 5 m and length 1.5 m with side slope of 1 : 2. The top width of the fill is 10.5 m. G = 2.7.



Solution:

given

$$\rho_{d(borrow)} = 1.78 \text{ g/cc.}$$

$$e_{(borrow)} = e_b = 0.63$$

$$G = 2.7$$

$$\text{Base width of highway fill} = 2(2 \times 5) + 10.5 = 30.5 \text{ m}$$

$$\text{Volume of highway fill} = \frac{1}{2}(10.5+30.5) \times 5 \times 1500 = 153750 \text{ m}^3$$

Dry density of highway fill (90%) = 0.90 × 1.78 = 1.602 g/cc

To find void ratio of fill:

We know

$$\rho_{d(fill)} = \frac{G\rho_w}{1+e_f}$$

$$1.602 = \frac{2.7 \times 1}{1+e_f}$$

$\therefore$ $e_f = 0.685$

Since volume of soil solid remain constant in both borrow pit and fill, Hence

$$\frac{V_b}{(1+e_b)} = \frac{V_f}{(1+e_f)}$$

$$V_b = \frac{(1+e_b)}{(1+e_f)} \times V_f = \frac{(1+0.63)}{(1+0.685)} \times 153750 = 57486 \text{ m}^3$$

Example 5.9 The in-situ void ratio of a granular soil deposit is 0.59. The void ratios in loosest and densest state were found to be 0.81 and 0.37 respectively. Determine the relative density and relative compaction of the soil deposit. $G = 2.7$.

Solution:

Given:

$e = 0.59$, $e_{max} = 0.81$, $e_{min} = 0.37$

$$\text{Relative density } (\% I_D) = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100\% = \frac{0.81 - 0.59}{0.81 - 0.37} \times 100 = 50\%$$

$$\gamma_{d(max)} = \frac{G\gamma_w}{1+e_{min}} = \frac{2.7 \times 9.81}{1+0.37} = 19.33 \text{ kN/m}^3$$

$$\gamma_{d(in\text{-}situ)} = \frac{G\gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.59} = 16.65 \text{ kN/m}^3$$

$$\text{Relative compaction} = \frac{\gamma_{d(in\text{-}situ)}}{\gamma_{d(max)}} = \frac{16.65}{19.33} \times 100 = 86.13\%$$

Example 5.10 In Indian standard light compaction test, the bulk unit weight of soil sample was found to be 18 kN/m³ at a moisture content of 13%. Determine

(i) degree of saturation of sample

(ii) additional moisture content required for complete saturation

(iii) maximum theoretical dry unit weight

$G = 2.68$

Solution:

(i) Given:

$$\gamma_t = \frac{G(1+w)}{1+\frac{wG}{S}}\gamma_w$$

$$18 = \frac{2.68(1+0.13) \times 9.81}{1+\frac{(0.13 \times 2.68)}{S}}$$

$$1+\frac{0.3484}{S} = 1.65$$

$$S = 0.5356 \text{ or } 53.56\%$$

(ii) Void ratio at 53.56% saturation

$$e = \frac{w \cdot G}{S} = \frac{0.13 \times 2.68}{0.5356} = 0.65$$

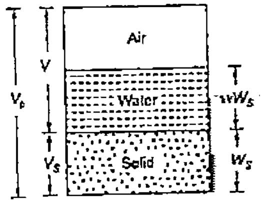
At a given compactive effort, void ratio remain constant. The water content at full saturation may be given as

$$w = \frac{1 \times e}{G} = \frac{0.65}{2.68} = 0.242 \text{ or } 24.20\%$$

$\therefore$ Additional water content required for complete saturation

$= 24.20 - 13 = 11.2\%$

Example 5.11 In the construction of a levee by compacting soil excavated from a borrow area to dry density of 1.78 g/cc, to make the compaction process more workable, an optimum water content of 15% is necessary. However the natural moisture content and bulk density of the soil were 9% and 1.83 g/cc respectively. Find out the quantity of water to be added for every 100 m³ of finished embankment.



Borrow area

Solution:

For embankment Given :

$$\gamma_{d(max)} = 1.78 \text{ g/cc} = 1.78 \text{ t/m}^3$$

$\therefore$ $W_s = 1.78 \times 100 = 178$ tonns

The weight of water needed in embankment

$$W_w = wW_s = 0.15 \times 178 = 26.7 \text{ ton}$$

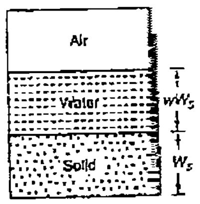
For borrow area given : $\gamma_t = 1.83$ g/cc $= 1.83$ t/m³

$w = 9\%$ or 0.09

$$\gamma_t = \frac{\text{weight of soil}}{\text{volume of soil}} = \frac{W_s + W_w}{V_b} = \frac{Ws(1+w)}{V_b}$$

$$\therefore \quad 1.83 = \frac{178(1+0.09)}{V_b}$$

$$V_b = 106.02 \text{ m}^3$$



Finished embankment 100(m³)

Weight of water present in the soil taken from borrow area,

$$W_w = wW_s = 0.09 \times 178 = 16.02 \text{ ton}$$

Therefore quantity of water to be added

$= 26.70 - 16.02 = 10.68$ ton

$$\therefore \text{Volume of water to be added} = \frac{W_w}{\rho_w} = \frac{10.68}{10^{-3}} = 10680 \text{ litres}$$

Summary

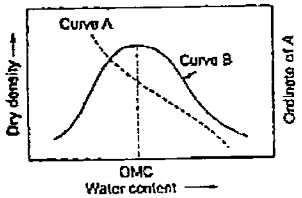
- Soil compaction is the process of increasing the density of the soil by applying some mechanical energy and thereby reducing air voids.
- Factors affecting the compaction of a soil are moisture content, compactive effort and method of compaction.
- Higher compactive effort increases the maximum dry unit weight and reduces the optimum water content.
- Comparison of properties of cohesive soil on dry and wet sides of OMC

Table : Comparison of dry of Optimum with Wet of Optimum Compaction

Property	Dry optimum	Wet of optimum
Structure after compaction	Flocculated (random)	Dispersed (oriented)
Water deficiency	More	Less
Permeability	More, isotropic	Less isotropic ($k_H > k_V$)
Compressibility at low stress at high stress	Low High	High Low
Swellability	High	Low
Shrinkage	Low	High
Stress strain behaviour	Brittle; high peak higher elastic modulus	Ductile; no peak lower elastic modulus
Construction pore water pressures	Low	High
Strength (undrained) as moulded after saturation	High somewhat higher if swelling prevent	Much lower low

Objective Brain Teasers

Q.1 Curve B shows the typical compaction curve of a soil in the Proctor test. Dotted curve A is shown superimposed on the same graph. Which one of the following expressions corresponds to the ordinate axis of curve A?



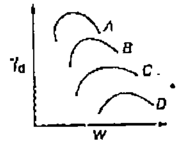
(a) Zero air voids
(b) Wet density
(c) Penetration resistance of Proctor needle
(d) 95% saturation

Q.2 At what value of saturation does the zero air voids curve in a compaction test represent the dry density?
(a) 0% (b) 80%
(c) 100% (d) 50%

Q.3 Which one of the following is the correct statements?
Vibratory rollers are suitable for compacting
(a) organic soil
(b) clays
(c) sands and gravels
(d) clayey silts

Q.4 Why are sheep foot rollers more effective in compacting clayey soils?
(a) There is differential expulsion of water under the roller
(b) Contact pressure is high
(c) Roller speed is high
(d) Drum width is large

Q.5 The results (curves A, B, C and D) of four compaction tests on different soils are shown in the graph:



1. Silty sand, modified test
2. Silty sand, standard test
3. Fat clay, modified test
4. Fat clay, standard test

Curves A, B, C and D correspond respectively to tests:
(a) 1, 3, 2 and 4 (b) 1, 2, 3 and 4
(c) 2, 1, 3 and 4 (d) 2, 1, 4 and 3

Q.6 Given below are methods of compaction:
1. Vibration technique
2. Flooding the soil
3. Sheep foot roller
4. Tandem roller
5. Heavy weights dropped from a height

The methods suitable for cohesionless soils include:
(a) 1, 2 and 3 (b) 2, 3 and 4
(c) 1, 2 and 5 (d) 3, 4 and 5

Q.7 Match List-I (Type of soils) List-II (Compaction parameters) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Sand	1. OMC = 18%, $\gamma_{dry\,max}$ = 17 kN/m^3
B. Sandy clay	2. OMC = 14%, $\gamma_{dry\,max}$ = 18.9 kN/m^3
C. Silty clay	3. OMC = 15%, $\gamma_{dry\,max}$ = 17.4 kN/m^3
D. Heavy clay	4. OMC = 10%, $\gamma_{dry\,max}$ = 20.5 kN/m^3

Codes:

	A	B	C	D
(a)	2	4	3	1
(b)	2	4	1	3
(c)	4	2	3	1
(d)	4	2	1	3

Q.8 Consider the following statements regarding the flow nets:
1. Flow lines and equipotential lines always intersect one-another at right angles irrespective of the permeability characteristics.
2. For an isotropic soil, the spacing of lines is inversely proportional to the hydraulic gradient.
3. For an anisotropic soil having greater horizontal coefficient of permeability, a flow net of approximate squares can be constructed by reducing the vertical dimensions of the flow domain to a certain scale.
4. For an isotropic soil having greater horizontal coefficient of permeability, a flow net of approximate squares can be constructed by reducing the vertical dimensions of the flow domain to a certain scale.

Which of these statements are correct ?
(a) 2, 3 and 4 (b) 2 and 3
(c) 1, 2 and 4 (d) 1 and 3

Q.9 A clear dry sand sample is tested in a direct shear test. The normal stress and the shear stress at failure are both equal to 120 kN/m^2. The angle of shearing resistance of the sand will be:
(a) 25° (b) 35°
(c) 45° (d) 55°

Q.10 Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I
A. Optimum moisture content
B. Vibratory rollers
C. Zero air void line

List-II
1. Compaction of cohesive soil
2. Compaction of granular soil
3. Maximum dry density
4. Relative density
5. 100% saturation

Codes:

	A	B	C
(a)	4	1	3
(b)	3	2	5
(c)	4	1	5
(d)	3	2	4

Q.11 If specific gravity of soil solids of a soil having optimum moisture content 16.5% and maximum dry density 1.57 g/cc is 2.65, void ratio at optimum moisture content is

(a) 0.588 (b) 0.688
(c) 0.788 (d) 0.888

Q.12 The maximum dry density upto which any soil can be compacted depends upon

(a) amount of compaction energy alone
(b) moisture content alone
(c) both compaction energy and moisture content
(d) none of these

Q.13 At optimum moisture content of soil mass, the soil saturation will be around

(a) 100% (b) 90 – 95%
(c) 0% (d) none of these

Q.14 The percentage of voids filled with air in a soil compacted under OMC, shall be around

(a) 0% (b) 5 – 10%
(c) 100% (d) none of these

Direction: The following items consists of two statements, one labelled as 'Assertion A' and the other as 'Reason R'. Select your answer to these items using the codes given below.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.15 Assertion (A): The process of compaction is accompanied by the expulsion of air.
Reason (R): The degree of compaction of a soil is characterized by its dry density.

Q.16 Assertion (A): For a given soil, the optimum moisture content increases with the increase in compactive effort.
Reason (R): Higher the compactive effort, higher is the dry density at the same moisture content.

Q.17 Consider the following:

1. Increase in shear strength and bearing capacity
2. Increase in slope stability
3. Decrease in settlement of soil
4. Decrease in permeability

Which of the above with respect to compaction of soil is/are correct:

(a) 1 only (b) 1 and 2 only
(c) 2 and 3 only (d) 1, 2, 3 and 4

Q.18 The insitu void ratio of a granular soil deposit is 0.50. The maximum and minimum void ratios of the soil were determined to be 0.75 and 0.35. G_s = 2.67 the relative density and relative compaction of the soil are respectively (γ_w = 9.8 kN/m^3)

(a) 62.5%, 89.9% (b) 89.9%, 62.5%
(c) 62.5%, 96.6% (d) 89.9%, 96.6%

Q.19 Compaction of an embankment is carried out in 400 mm thick layers. The rammer used for compaction has a foot area of 0.1 m^2 and the energy imparted in every drop of rammer is 350 N-m. Assuming 40% more energy in each pass over the compacted area due to overlap, the number of passes required to develop compactive energy equivalent to Indian Standard light compaction for each layer would be

(a) 26 (b) 33
(c) 49 (d) 54

Q.20 Assertion (A): Road built on black cotton soils show crack after some time.
Reason (R): Black cotton soils settle and this results in deformation.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Answers

1. (c)	2. (c)	3. (c)	4. (b)	5. (b)
6. (c)	7. (c)	8. (d)	9. (c)	10. (b)
11. (b)	12. (c)	13. (b)	14. (b)	15. (b)
16. (d)	17. (d)	18. (a)	19. (d)	20. (a)

Hints and Explanations:

4. (b)
Under sheep foot rollers the contact pressure is very high and this results in kneading action which is responsible for effective compaction of clayey soils

5. (b)
With the increase of fine particles in sand, the OMC decreases and maximum dry density increases upto certain extent. Thereafter any further increase in fine particles will lead to increase of OMC and decrease of maximum dry density. Further in modified test, the OMC will be less and maximum dry density will be more compared to that in standard test. Therefore combining both:

OMC 1 < 2 < 3 < 4

Maximum dry density

1 < 2 < 3 < 4

So A-1, B-2, C-3, D-4

6. (c)
Cohesionless soils can be compacted by vibration; flooding and dropping heavy weight from height. Kneading and tamping actions provided by sheep foot roller and tandem roller are not effective for cohesionless soils.

7. (c)
Heavy (fat) clay means pure clay.
As the fine particle content is increased in a cohesionless soil, the dry density achieved is very high compared to that in cohesionless soil. However when fine particles increase beyond a limit, the dry density decrease and OMC increases.

Thus OMC of D > C > B > A

So D – 1, C – 3, B – 2, A – 4

Maximum dry density of A > B > C > D

So A – 4, B – 2, C – 3, D – 1

8. (d)
The flow net is governed by velocity potential function and stream function.

$$\frac{\partial\phi}{\partial x} = k_x;\ \frac{\partial h}{\partial x} = v_x\ \&\ \frac{\partial\phi}{\partial z} = k_z;\ \frac{\partial h}{\partial z} = v_z$$

$$\text{and } \frac{\partial\psi}{\partial z} = v_x, \frac{\partial\psi}{\partial z} = -v_z$$

The flow lines and equipotential lines are thus always orthogonal to each other.
The line will be spaced proportional to the hydraulic gradient.
For square mesh potential drop is same throughout the field.
For anisotropic soil, the horizontal dimension of flow domain is reduced by $\sqrt{k_x/k_z}$ factor.

Thus statements (1) and (3) are correct.

9. (c)

$\tau = \sigma_n \tan\phi$ given $\tau = \sigma_n = 120\text{kN/m}^2$

$\therefore \tan\phi = 1$ $\phi = 45°$

10. (b)
A – 3; B – 2; C – 5

11. (b)
given

$$\gamma_d = 1.57$$
$$w = 0.165$$
$$G = 2.65$$

using, $$\gamma_d = \frac{G \cdot \gamma_w}{1+e}$$

$$1.57 = \frac{2.65 \times 1}{1+e}$$

$$1 + e = 1.688$$

$$\therefore\ e = 0.688$$

14. (b)
%Air voids = 1 – S = 5 – 10%

18. (a)
Relative density

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} \times 100\%$$

$$= \frac{0.75 - 0.50}{0.75 - 0.35} \times 100 = 62.5\%$$

$$\gamma_{d(max)} = \frac{G_s\gamma_w}{1 + e_{min}}$$

$$= \frac{2.67 \times 9.8}{1 + 0.35} = 19.38 \text{ kN/m}^3$$

$$\gamma_{d(min)} = \frac{G_s\gamma_w}{1+e_{max}}$$

$$= \frac{2.67 \times 9.8}{1+0.75} = 14.95 \text{ kN/m}^3$$

$$\gamma_{d(insitu)} = \frac{G_s\gamma_w}{1+e_{insitu}}$$

$$= \frac{2.67 \times 9.8}{1+0.50} = 17.44 \text{ kN/m}^3$$

∴ Relative compaction,

$$\frac{\gamma_{d(insitu)}}{\gamma_{d(max)}} = \frac{17.44}{19.38} \times 100 = 89.9\%$$

19. **(d)**

As per Indian Standard light compaction test, a hammer of 2.6 kg is allowed to fall from a height of 310 mm and 3 layers are tamped 25 times in a mould of volume 1000 cc.

∴ Energy imparted

$$= 2.6 \times 25 \times 3 \times 9.81 \times \frac{310}{1000}$$

$$= 593.01 \text{ N-m}$$

For embankment compaction, the volume of soil covered by rammer

$$= 0.1 \times 10^4 \times \frac{400}{10} = 40000 \text{ cc}$$

∴ Energy imparted in every drop of hammer

$$= \left(1+\frac{40}{100}\right) \times 350 = 490 \text{ N-m}$$

If 'n' number of passes are required to develop equivalent energy to Indian Standard light compaction test, then

$$n \times \frac{490}{40000} = \frac{593.01}{1000}$$

$$\Rightarrow \quad n = 48.41 \approx 49 \text{ (approx.)}$$

Student's Assignments

Q.1 The total unit weight (γ_t) of soil is 18.8 kN/m^3, the specific gravity (G) of the solid particles of the soil is 2.67 and the water content (w) of the soil is 12%. Calculate the dry unit weight (γ_d), void ratio (e) and the degree of saturation.

Ans. γ_d = 16.79 kN/m^3, e = 0.56 and S = 57.2%

Q.2 Embankment for a highway 30 m wide and 1.5 m is to be constructed from a sandy soil trucked from a borrow pit. The water content of the sandy soil in the borrow pit is 15% and its void ratio is 0.69. The specification requires the embankment be compacted to a dry unit weight of 18 kN/m^3. Determine, for 1 km length of embankment, the following:

(i) The volume of sandy soil from the borrow pit required to construct the embankment.

(ii) The number of trucks taking 10 m^3 of volume of soil required for the construction.

(iii) The weight of water per truck load of sandy soil.

(iv) The degree of saturation of the sandy soil in-situ.

Ans. V_b = 51.6 × 10^3 m^3, N = 5160 trucks

W_w/Truck = 23.6 kN, S = 59%

Q.3 Some soil has been dumped loosely from a scraper. It has a unit weight of 16 kN/m^3, a water content of 10.5% and a specific gravity of solids of 2.68. Find the void ratio, porosity and unit weight of the soil in the loose condition.

To make the compaction process more workable, an optimum water content of 15% is necessary. How much of water should be added in liters per cubic meter of soil to raise the water content to the optimum?

Ans. Loose condition: ρ_d = 1.468 g/cc

and γ_c = 14.40 kN/m^3

Water to be added = 51.4 kg

Q.4 A sample of soil was prepared by mixing a quantity of dry soil with 10% by mass of water. Find the mass of this wet mixture required to produce a cylindrical, compacted specimen of 15 cm diameter and 12.5 cm high, having 6% air content. Find also the void ratio and the dry density of the specimen if G = 2.68.

Ans. M = 460.65 g, e = 0.285,

ρ_d = 2.085 g/cc

Q.5 The soil from a borrow area having an average in-situ unit weight of 15.5 kN/m^3 and water content of 10%, was used for the construction of an embankment of total finished volume 6000 m^3. In half of embankment, due to improper control during rolling, the dry unit weight achieved was slightly lower. If the dry unit weight in the two parts is 16.5 kN/m^3 and 16.0 kN/m^3, find the volume of borrow area soil used in each part.

Ans. 3512.9 m^3 and 3.406.45 m^3

■■■■

CHAPTER 06

Principle of Effective Stress, Capillarity and Permeability

6.1 Introduction

Terzaghi was the first to enunciate the effective stress principle. It can be said that in doing that, he opened the flood gates to the discipline of soil mechanics. Some of the basic developments on shear strength, compressibility and lateral earth pressures are a direct offshoot of the effective stress concept. The effective stress principle consists of two parts:

(i) Definition of the effective stress

(ii) Importance of effective stress in engineering behavior of soil.

6.2 Total Stress, Pore Pressure and Effective Stress

6.2.1 Total Stress

Normal stress is defined as the sum of the normal component of the forces (ΣN) over a plane ($X-X$) divided by the area of plane (A). Generally, it is denoted by σ.

$$\therefore \qquad \sigma = \frac{\Sigma N}{A}$$

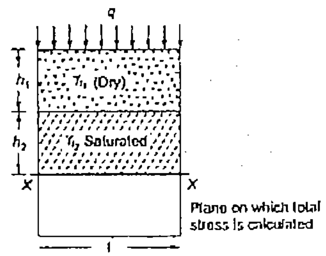


Fig.6.1

Let us consider a stratified soil sample of unit width and unit length for the plane over which normal stresses to be computed.

$$\therefore \qquad \sigma = \frac{q \cdot A + \gamma_{t_1} \times h_1 \times A + \gamma_{t_2} \times h_2 \times A}{A}$$

$$= q + \gamma_d h_1 + \gamma_{sat} h_2$$

If external pressure q is zero, then total stress caused due to the over burden alone are known as 'Geostatic stress'.

$$\text{then,} \qquad \sigma = \gamma_d h_1 + \gamma_{sat} h_2 \qquad \text{...(i)}$$

Remember: Total stress is a parameter which can be measured with suitable instrument, such as a pressure cell.

6.2.2 Pore Pressure

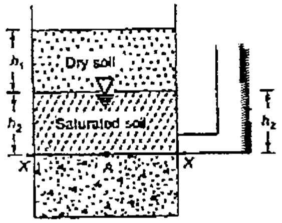


Fig.6.2

It is the pressure exerted by the pore water filled in the void of the soil skeleton. It is denoted by u and equal to the depth (h_2) below the ground water table upto the point (A) where it is measured and multiplied by the unit weight of water γ_w.

$$\therefore \qquad u = h_2\gamma_w \qquad \text{...(ii)}$$

Pore pressure is also called the neutral stress because it acts on all sides of the particle, hence does not cause the soil particles to press against adjacent particles.

Remember: Pore water pressure, like the total stress is also a measurable parameter. A standpipe, piezometer or pore water pressure transducer may be used to measure the pore water pressure.

6.2.3 Principle of Effective Stress

Total stress (σ) is made up of two parts

(i) one part is due to pore water pressure (u)

(ii) the other part is due to pressure exerted by the soil skeleton which is called effective stress ($\bar{\sigma}$).

$$\text{i.e.} \qquad \sigma = u + \bar{\sigma}$$

$$\therefore \qquad \bar{\sigma} = \sigma - u$$

Substituting value of σ and u from equation (i) and (ii), we have

$$\bar{\sigma} = (\gamma_d h_1 + \gamma_{sat} h_2) - \gamma_w h_2$$

$$= \gamma_d h_1 + (\gamma_{sat} - \gamma_w) h_2 = \gamma_d h_1 + \gamma' h_2$$

where γ' is submerged unit weight.

Remember

- Effective stress in soils is a grain to grain contact pressure which a soil particle can exhibit.
- The effective stress is not a physical parameter and cannot be measured.
- Increase in effective stress causes the particles to pack more closely, decreases the void ratio, leads to a decrease in compressibility and increases in the shearing resistance of the soil.
- When there is an equal increase in the total stress and the pore pressure, then effective stress remain unchanged.
- Principle of effective stress applies only to normal stresses not to the shear stress.

6.3 Effective Stress in Partially Saturated Soils

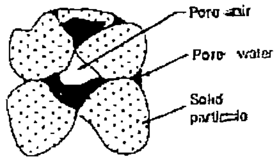


Fig.6.3

Effective stress, $\bar{\sigma} = \sigma - u_a + \chi(u_a - u_w)$

where,
σ = Total stress
u_a = Pore air pressure
u_w = Pore water pressure
χ = The parameter to be determined experimentally
= 0 (for dry soil)
= 1 (for saturated soil)

It means that, if the soil is partially saturated, pore air pressure is considered along with pore water pressure to analyze the stresses in the soil mass.

6.4 Capillarity in Soils

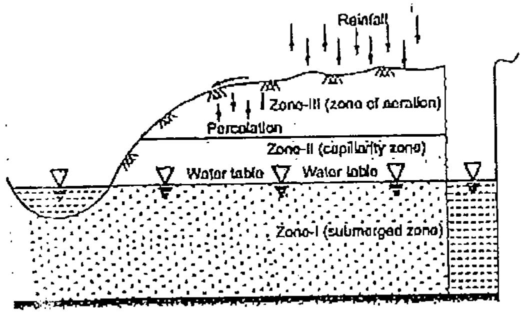


Fig. 6.4 Zones of soil water below ground surface

As we know, the rain water percolates into the ground under the influence of gravity and gets stored in the soil pores, over an impervious stratum, in the form of ground water reservoir.

The upper surface of the zone of full saturation of the soil is called the water table or phreatic surface. At the water table, the ground water is subjected to atmospheric pressure. In other words, the pore pressure is zero.

Soil water can exist in three zones as shown below:

- **Zone-I (Submerged Zone):** This zone exist below the ground water table. The soil in this zone is in submerged condition and pore pressure is hydrostatic i.e. positive.
- **Zone-II (Capillarity Zone):** If gravity was the only force, acting on the percolating water and taking it downward, then the soil above the water table would be completely dry. But it is not so in actual practice. Soil in this zone is completely saturated upto some height above the water table. This phenomenon of rising water in soil is known as capillarity in soils.
- **Zone–III (Zone of Aeration):** Above the capillarity zone and upto a certain height, there exist a zone called the zone of aeration, in which the soil is able to retain small water droplets, surrounded on all sides by air. This water is known as contact moisture or hygroscopic water. The contact moisture is retained by the soil against the gravity drainage by the action of capillary.

6.4.1 Capillary Rise in Soils

We can get an understanding of capillarity in soil by idealizing the continuous void spaces as capillary tubes. Consider a single idealized tube as shown in figure below.

The surface tension (force) pulls the water upward till the height h_c, at which the weight of water in the column is in equilibrium with the magnitude of the surface tension force.

For equilibrium

$$h_c \cdot \frac{\pi d^2}{4}\gamma_w = (\pi \cdot d)T\cos\alpha$$

$$h_c = \frac{4T\cos\alpha}{\gamma_w \cdot d}$$

where, α = contact angle

T = surface tension

d = diameter of capillary tube

γ_w = unit weight of water

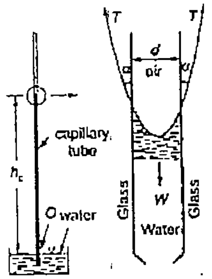


Fig. 6.5 Capillary Rise in Soils

Remember

- $h_c(\text{cm}) = \dfrac{0.3084}{d(\text{cm})}$ ....at 4°C
- $h_c(\text{cm}) = \dfrac{0.2975}{d(\text{cm})}$ ....at 20°C

Because of the complex nature of the soil, a theoretical prediction of capillary rise in soil is not possible. Terzaghi and Peck suggested an approximate relationship to find capillary rise in-situ.

$$h_c(\text{cm}) = \frac{C}{eD_{10}}$$

where, C = Empirical constant: the value of which depends on the shape and surface impurities of the grains and lies between 0.1 and 0.5 cm².

e = void ratio

D_{10} = Effective grain size

Do you know? Capillarity involves both adhesive and cohesive forces.

6.4.2 Capillary Pressure

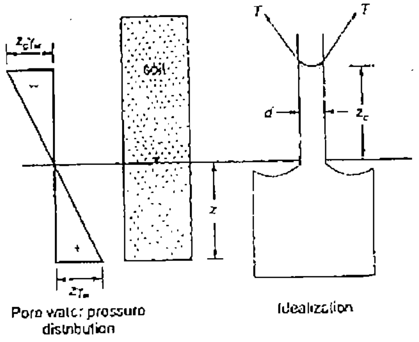


Fig. 6.6

Capillary water rises against gravity and is held by the surface tension. Therefore the capillary water exerts a tensile force on soil and resulting negative pressure of water (Capillary pressure) creates attraction between the particles.

From the effective stress equation,

$$\bar{\sigma} = \sigma - u$$

When the pore water is in compression, as in case of hydrostatic pressure, u is taken positive whereas when it is in tension, u is taken negative.

Therefore effective stress in capillary zone

$$\bar{\sigma} = \sigma - (-u_c) = \sigma + u_c$$

It means that due to capillarity in soils, effective stress increases.

Do You Know

- Negative pressure of water (capillary pressure) held above water table results in attractive forces between particles. It is called as soil suction.
- Bulking of sand also occurs because of capillarity. Capillarity produces apparent cohesion which holds the particles in cluster, enclosing honey combs.
- Water may flow over the crest of an impermeable core in a dam even though, the free surface may be lower than the crest of core. This effect of capillarity is known as capillary siphoning.

Example 6.1 Compute the maximum capillarity tension for a tube of 0.12 mm in diameter at 4°C.

Solution:

Using $h_c = \dfrac{0.3084}{d(\text{cm})} = \dfrac{0.3084}{0.012}$

$= 25.7$ cm or 0.257 m

$\therefore$ Capillary Tension $= h_c \gamma_w$

$= 0.257 \times 9.81$

$= 2.521$ kN/m²

Example 6.2 The capillary rise in a soil having $D_{10} = 0.07$ mm is 49 cm. Estimate the capillary rise in another soil having its $D_{10} = 0.12$ mm. Assume the same void ratios in both the soils.

Solution:

Soil I	Soil II
$D_{10} = 0.07$ mm	$D'_{10} = 0.12$ mm
$= 0.007$ cm	$= 0.012$ cm
$h_c = 49$ cm	$h'_c = ?$

Using $h_c \cdot D_{10} = \text{Constant}$

$\therefore$ $h_c \cdot D_{10} = h'_c \cdot D'_{10}$

$$h'_c = \left(\frac{D_{10}}{D'_{10}}\right) h_c$$

$$= \left(\frac{0.017}{0.012}\right) \times 49 = 28.58 \text{ cm}$$

Hence, the capillary rise in the second soil would be 28.6 cm.

6.5 Geostatic Stresses in Soils

Stresses within the soil mass may be caused by the self weight of the soil and also by the external loads which may be applied to the soil. The pattern of stresses caused by the external loads is usually complicated one. However, there is one common situation in which the self-weight of soil gives rise to a very simple pattern of stresses, that is when the ground surface is horizontal and the nature of soil does not very significantly in the horizontal direction. Such a situation frequently occurs in the case of sedimentary deposits. The total stress caused in such a situation at a point in a soil mass is called the geostatic stress.

Do You Know

- In the geostatic situation, there are no shear stresses upon the horizontal or vertical planes within the soil.
- If external loading contributes to this stress, the stress will simply be called as the total stress.

The vertical geostatic stress at a point below the surface is equal to the weight of the soil lying directly above the point.

If the unit weight of the soil (γ) is constant with depth, then

Total stress $= \sigma$

$= \gamma \times (z \times 1 \times 1)$

$= \gamma \cdot z$

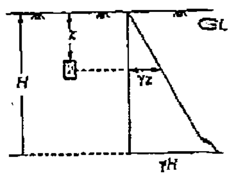


Fig. 6.7 Uniform Soil

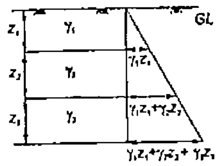


Fig. 6.8 Stratified soil

If the soil is stratified with different unit weights for each stratum as shown in figure below:

Then σ can be calculated as

$$\sigma = \gamma_1 z_1 + \gamma_2 z_2 + \gamma_3 z_3 + \ldots\ldots$$

$$= \Sigma \gamma z$$

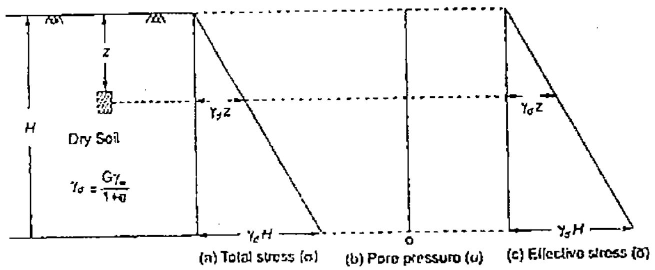
Case 1: When Soil Is Dry

Consider a situation when the water table is at large distance from the ground surface. Then, the stresses at a depth z is given as

Total stress, $\sigma = \gamma_d z$

Pore water pressure, $u = 0$

and effective stress, $\bar{\sigma} = \sigma - u = \gamma_d z - 0 = \gamma_d z$



(a) Total stress (σ) (b) Pore pressure (u) (c) Effective stress ($\bar{\sigma}$)

Fig. 6.9 Vertical stress distribution in dry soil

Case 2: When Soil Is Moist

There is a situation of partially saturated soil wherein it is difficult to predict the pore water pressure distribution. In this case pore pressure is neglected. Bulk unit weight of soil is calculated by

$$\gamma_t = \frac{(G + Se)}{1 + e} \gamma_w$$

Thus,

Total stress, $\sigma = \gamma_t \cdot z$

Pore water pressure, $u = 0$

$\bar{\sigma} = \sigma - u = \gamma_t z - 0 = \gamma_t \cdot z$

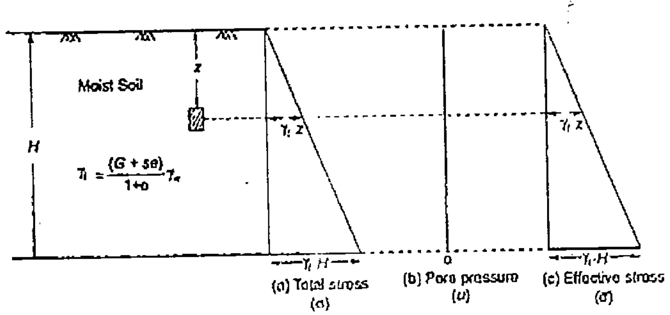


Fig. 6.10 Vertical stress distribution in moist soil

Case 3: When Soil Is Saturated

This is a situation of saturated soil above water table, which is due to some reasons like rain, watering or irrigation (except capillarity of soil). Hence pore water pressure is zero. Therefore, the stresses are governed by the saturated weight of the soil.

Thus,

Total stress, $\sigma = \gamma_{sat} \cdot z$

Pore water pressure, $u = 0$

Effective stress, $\bar{\sigma} = \sigma - u = \gamma_{sat} \cdot z - 0 = \gamma_{sat} \cdot z$

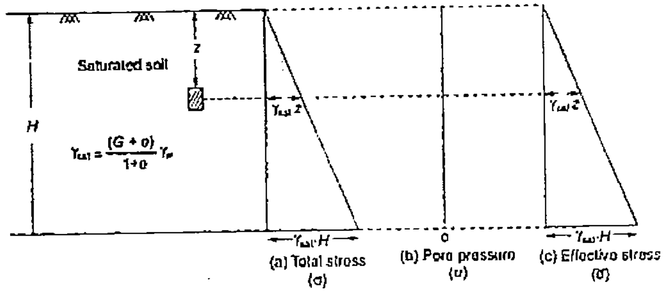


Fig. 6.11 Vertical stress distribution in saturated soil

Case 4: When Soil Is Completely Submerged

In this situation, soil is under the water table in submerged condition with water table at ground level. Therefore pore water pressure is hydrostatic. In this case, total stress is governed by saturated unit weight of soil and effective stress is governed by submerged unit weight.

Thus,

Total stress, $\sigma = \gamma_{sat} \cdot z$

Pore water pressure, $u = \gamma_w \cdot z$

Effective stress, $\bar{\sigma} = \sigma - u = \gamma_{sat} \cdot z - \gamma_w \cdot z = \gamma' z$

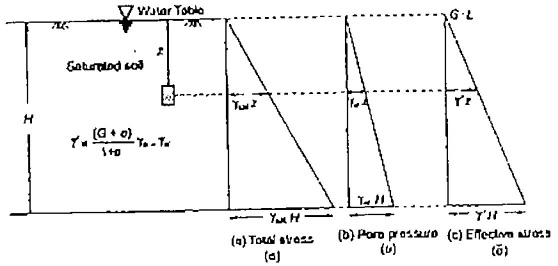


Fig. 6.12 Vertical stress distribution in submerged soil

Case 5: When Soil Is Completely Saturated by Capillarity

In this situation, soil is completely saturated but pore water pressure is negative and vary linearly from zero at water table to $\gamma_w(H-z)$ at any depth below ground level.

Total stress, $\sigma = \gamma_{sat} \cdot z$

Pore pressure, $u = -\gamma_w(H-z)$

Effective stress, $\bar{\sigma} = \sigma - u = \gamma_{sat} \cdot z + \gamma_w(H-z) = (\gamma_{sat} - \gamma_w)z + \gamma_w H$

$= \gamma' z + \gamma_w H$

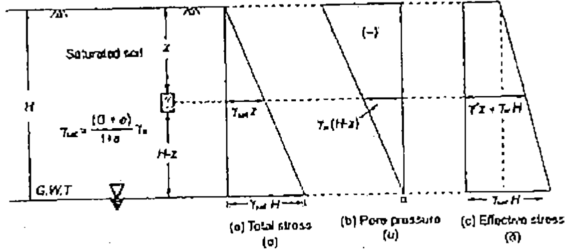


Fig. 6.13 Vertical stress distribution in saturated soil by capillarity

Special Case

When height of capillary rise is less than H (i.e. $0 < h_c < H$)

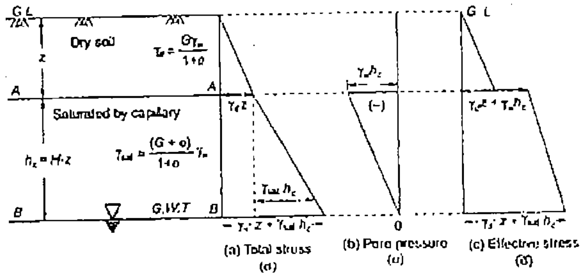


Fig. 6.14 Vertical stress distribution in partially saturated soil

At Ground Level

Total stress, $\sigma = 0$

Pore water pressure, $u = 0$

Effective stress, $\bar{\sigma} = 0$

At the Level of *A-A*

Total stress, $\sigma_A = \gamma_d \cdot z$

Pore water pressure, $u_A = -\gamma_w h_c$

Effective stress, $\bar{\sigma} = \sigma - u = \gamma_d \cdot z - (-\gamma_w h_c) = \gamma_d \cdot z + \gamma_w h_c$

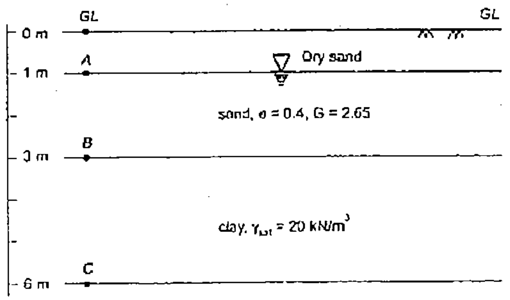
At the Level of *B-B*

Total stress, $\sigma_B = \gamma_d \cdot z + \gamma_{sat} h_c$

Pore water pressure, $u_B = 0$

Effective stress, $\bar{\sigma} = \sigma - u = \gamma_d \cdot z + \gamma_{sat} h_c$

Example 6.3 For the subsoil condition shown in figure below, calculate the total, neutral and effective stress at 1 m, 3 m and 6 m below the ground level. Assume $\gamma_w = 10$ kN/m³.



Solution:

Note that sand above ground water table is dry and completely saturated below the water table.

Therefore, we have to first determine γ_d and γ_{sat} for sand, using $e = 0.4$ and $G = 2.65$.

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 10}{1+0.4} = 18.93 \text{ kN/m}^3$$

Also,

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.4)\times 10}{1+0.4} = 21.79 \text{ kN/m}^3$$

(i) Total, neutral and effective stress at 1 m depth

(a) Total stress, $\sigma_A = \gamma_d \cdot z = 18.93 \times 1 = 18.93$ kN/m³

(b) Neutral stress, $u_A = 0$

(c) Effective stress, $\bar{\sigma}_A = \sigma_A \cdot u_A = 18.93 - 0 = 18.93$ kN/m²

(ii) Total, neutral and effective stress at 3 m depth

(a) Total stress, $\sigma_B = \gamma_d \times 1 + \gamma_{sat,\,sand} \times 2 = 18.93 \times 1 + 21.79 \times 2 = 62.51$ kN/m²

(b) Neutral stress, $u_B = \gamma_w \times 2 = 10 \times 2 = 20$ kN/m²

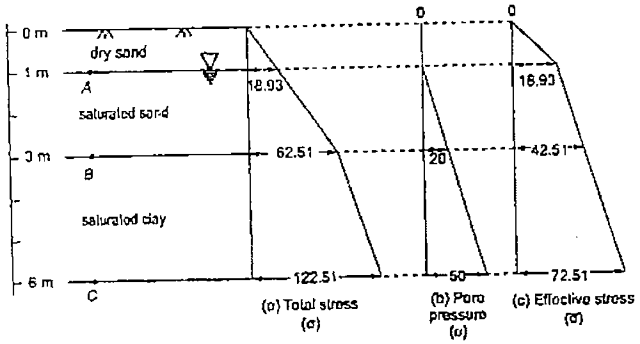
(c) Effective stress, $\bar{\sigma}_B = \sigma_B - u_B = 62.51 - 20 = 42.51$ kN/m²

(iii) Total, neutral and effective stress at 6 m depth

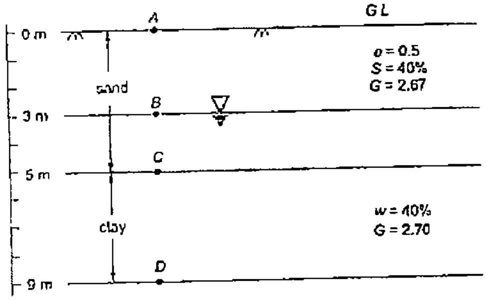
(a) Total stress, $\sigma_C = \gamma_d \times 1 + \gamma_{sat,\,sand} \times 2 + \gamma_{sat,\,clay} \times 3 = 18.93 \times 1 + 21.79 \times 2 + 20 \times 3 = 122.51$ kN/m²

(b) Neutral stress, $u_C = \gamma_w \times 5 = 10 \times 5 = 50$ kN/m²

(c) Effective stress, $\bar{\sigma}_C = \sigma_C - u_C = 122.51 - 50 = 72.51$ kN/m²



Example 6.4 For the subsoil condition shown in figure. Draw the total, neutral and effective stress diagram upto the depth of 9 m, neglecting the capillary rise.



Solution:

For Sand

Given: $e = 0.5$, $S = 0.40$, $G = 2.67$

Using

$$\gamma_t = \frac{(G+Se)\gamma_w}{1+e} = \frac{(2.67+0.4\times 0.5)\times 9.81}{1+0.5} = 18.77 \text{ kN/m}^3$$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.67+0.5)\times 9.81}{1+0.5} = 20.73 \text{ kN/m}^3$$

For Clay

Given: $w = 0.40$, $G = 2.70$

Clay is located well below the water table. Therefore, $S = 100\%$

Using $e = \frac{w \cdot G}{S} = \frac{0.40 \times 2.70}{1} = 1.08$

$\therefore \gamma_{sat,\, clay} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.7+1.08)}{1+1.08} \times 9.81 = 17.83 \text{ kN/m}^3$

(i) Total, neutral and effective stress at GL

Total stress, $\sigma_A = 0$

Neutral stress, $u_A = 0$

Effective stress, $\bar{\sigma}_A = \sigma_A - u_A = 0$

(ii) Total, neutral and effective stress at 3 m

Total stress, $\sigma_B = \gamma_t \times 3 = 18.77 \times 3 = 56.31 \text{ kN/m}^2$

Neutral stress, $u_B = 0$

Effective stress, $\bar{\sigma}_B = \sigma_B - u_B = 56.31 - 0 = 56.31 \text{ kN/m}^2$

(iii) Total, neutral and effective stress at 5 m

Total stress, $\sigma_C = \gamma_t \times 3 + \gamma_{sat,\, sand} \times 2 = 18.77 \times 3 + 20.73 \times 2 = 97.77 \text{ kN/m}^2$

Neutral stress, $u_C = \gamma_w \times 2 = 9.81 \times 2 = 19.62 \text{ kN/m}^2$

Effective stress, $\bar{\sigma}_C = \sigma_C - u_C = 97.77 - 19.62 = 78.15 \text{ kN/m}^2$

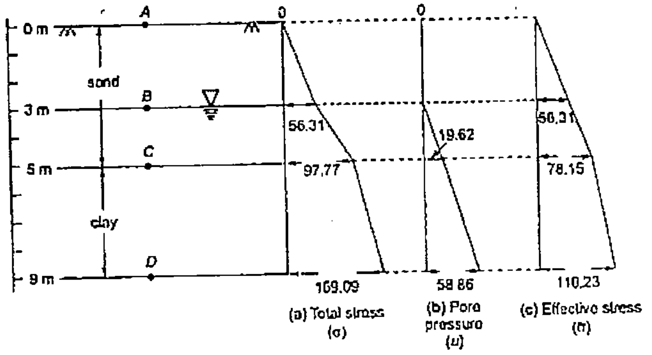
(iv) Total, neutral and effective stress at 9 m

Total stress, $\sigma_D = \gamma_t \times 3 + \gamma_{sat,\, sand} \times 2 + \gamma_{sat,\, clay} \times 4$

$= 18.77 \times 3 + 20.73 \times 2 + 17.83 \times 4 = 169.09 \text{ kN/m}^2$

Neutral stress, $u_D = \gamma_w \times 6 = 9.81 \times 6 = 58.86 \text{ kN/m}^2$

Effective stress, $\bar{\sigma}_D = \sigma_D - u_D = 169.09 - 58.86 = 110.23 \text{ kN/m}^2$

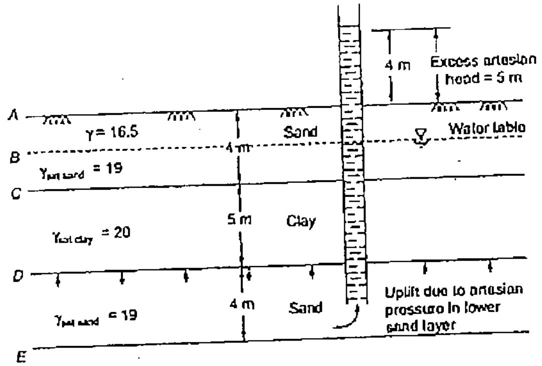


Variation of total, neutral and effective stress

Example 6.5 A 5 m thick clay layer lies between two layers of sand each 4 m thick, the top of the upper layer of sand being at ground level. The water table is 2 m below the ground level but the lower layer of sand is under artesian pressure, the piezometric surface being 4 m above ground level. The saturated unit weight of clay is 20 kN/m³ and that of sand is 19 kN/m³ above the water table unit weight of sand is 16.5 kN/m³. Calculate (a) the effective vertical stress at the top and bottom of the clay layer (b) Also draw the total vertical stress diagram in the given soil layers.

Solution:

Given conditions are shown in figure



(a) Effective stress at top clay layer

Total stress at C, $\sigma_C = \gamma \times z + \gamma_{sat,\, sand} \times 2$

$= 16.5 \times 2 + 19 \times 2 = 71 \text{ kN/m}^2$

Neutral stress at C, $u_C = \gamma_w \times 2 = 9.81 \times 2 = 19.62 \text{ kN/m}^2$

$\therefore$ Effective stress at C, $\bar{\sigma}_C = \sigma_C - u_C = 71 - 19.62 = 51.38 \text{ kN/m}^2$

Alternatively

Effective stress at C, $\bar{\sigma}_C = \gamma \times 2 + \gamma' \times 2$

$= 16.5 \times 2 + (19 - 9.81) \times 2 = 51.38 \text{ kN/m}^2$

Effective stress at bottom of clay layer

Total stress at D, $\sigma_D = \gamma \times z + \gamma_{sat,\, sand} \times 2 + \gamma_{sat,\, clay} \times 5$

$= 16.5 \times 2 + 19 \times 2 + 20 \times 5 = 171 \text{ kN/m}^2$

Neutral stress at D, $u_D = \gamma_w \times \text{Total water head} = 9.81 \times 13 = 127.53 \text{ kN/m}^2$

$\therefore$ Effective stress at D, $\bar{\sigma}_D = \sigma_D - u_D = 171 - 127.53 = 43.47 \text{ kN/m}^2$

Alternatively

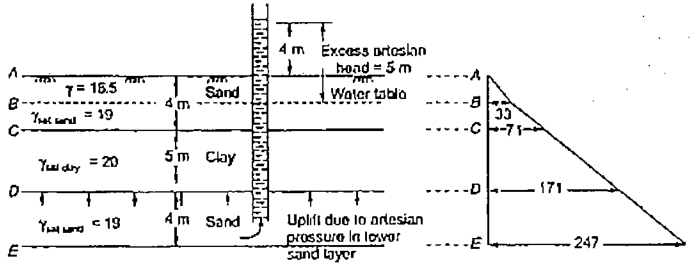
Effective stress at D, $\bar{\sigma}_D = \gamma \times 2 + \gamma_{sub,\, sand} \times 2 + \gamma_{sub,\, clay} \times 5 - \text{uplift caused by excess artesian head}$

$= 16.5 \times 2 + (19 - 9.81) \times 2 + (120 - 9.81) \times 5 - 9.8 \times 6$

$= 33 + 18.38 + 50.95 - 58.86$

$= 43.47 \text{ kN/m}^2$

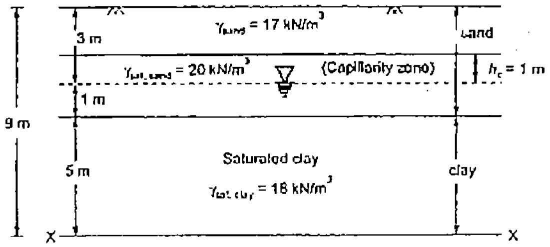
(b) Total stress diagram



Example 6.6 A layer of saturated clay 5 m thick is overlain by sand 4.0 m deep. The water table is 3 m below the top surface. The saturated unit weights of clay and sand are 18 kN/m^3 and 20 kN/m^3 respectively. Above the water table, the unit weight of sand is 17 kN/m^3. Calculate the effective pressures on a horizontal plane at a depth of 9 m below the ground surface. What will be the increase in the effective pressure at 9 m, if the soil gets saturated by capillary, upto height of 1 m above the water table?

Solution:

The given condition is shown in figure



(a) Effective stress at X - X

Total stress at X - X, $\sigma_X = \gamma_{sand} \times 3 + \gamma_{sat,sand} \times 1 + \gamma_{sat,clay} \times 5$

$= 17 \times 3 + 20 \times 1 + 18 \times 5 = 161$ kN/m^2

Neutral stress at X - X, $u_X = \gamma_w \times 6 = 9.81 \times 6 = 58.86$ kN/m^2

$\therefore$ Effective stress at X - X,

$\bar{\sigma}_X = \sigma - u = 161 - 58.86 = 102.14$ kN/m^2

Alternatively

$\bar{\sigma}_X = \gamma_{sand} \times 3 + \gamma_{sub,sand} \times 1 + \gamma_{sat,clay} \times 5$

$= 17 \times 3 + (20 - 9.81) \times 1 + (18 - 9.81) \times 5$

$= 51 + 10.19 + 8.19 \times 5$

$= 102.14$ kN/m^2

(b) When soil gets saturated upto height of 1 m above the water table by capillarity, then

Total stress at X - X, $\sigma = \gamma_{sand} \times 2 + \gamma_{sat,sand} \times 2 + \gamma_{sat,clay} \times 5$

$= 17 \times 2 + 20 \times 2 + 18 \times 5 = 164$ kN/m^2

Pore pressure a X - X, $u = \gamma_w \times 6 = 9.81 \times 6 = 58.86$ kN

$\therefore$ Effective stress at X - X,

$\bar{\sigma} = \sigma - u = 164 - 58.86 = 105.14$ kN/m^2

Alternatively

$\bar{\sigma} = \gamma_{sand} \times 2 + \gamma_{sat,sand} \times 1 + \gamma_{sub,clay} \times 1 + \gamma_{sub,clay} \times 5$

$= 17 \times 2 + 20 \times 1 + (20 - 9.81) \times 1 + (18 - 9.81) \times 5$

$= 34 + 20 + 10.19 + 8.19 \times 5$

$= 105.14$ kN/m^2

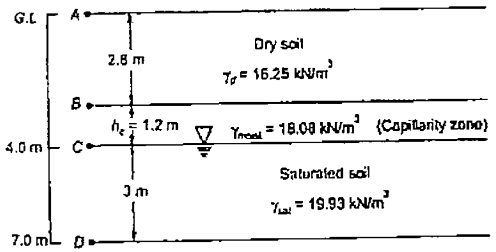
$\therefore$ Increase in pressure $= 105.14 - 102.14 = 3$ kN/m^2

Example 6.7 Granular soil deposit is 7 m deep over an impermeable layer. The ground water table is 4 m below the ground surface. The deposit has a zone of capillary rise of 1.2 m with a saturation of 50%, plot the variation of total stress, pore water pressure and effective stress with the depth of deposit, $e = 0.6$ and $G = 2.65$.

Solution:

The given situation is shown in figure.

Assume that soil above capillary zone is completely dry.



Unit weight of dry soil above water table,

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.6} = 16.25 \text{ kN/m}^3$$

Unit weight of moist soil in capillary zone,

$$\gamma_{moist} = \frac{(G+se)}{1+e}\gamma_w = \frac{(2.65 + 0.50 \times 0.6) \times 9.81}{1+0.6} = 18.08 \text{ kN/m}^3$$

Unit weight of completely saturated soil,

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.6) \times 9.81}{1+0.6} = 19.93 \text{ kN/m}^3$$

(a) Total stresses

(i) Total stress at A, $\sigma_A = 0$

(ii) Total stress at B, $\sigma_B = \gamma_d \times 2.8 = 16.25 \times 2.8 = 45.5$ kN/m^2

(iii) Total stress at C, $\sigma_C = \gamma_d \times 2.8 + \gamma_{moist,\,cap} \times 1.2$
$= 16.25 \times 2.8 + 18.08 \times 1.2 = 67.196 \text{ kN/m}^2$

(iv) Total stress at D, $\sigma_D = \gamma_d \times 2.8 + \gamma_{moist,\,cap} \times 1.2 + \gamma_{sat} \times 3$
$= 16.25 \times 2.8 + 18.08 \times 1.2 + 19.93 \times 3 = 126.986 \text{ kN/m}^2$

(b) Pore pressure

(i) Pore pressure at A, $u_A = 0$

(ii) Pore pressure at B, $u_B = -h_c \times \gamma_w = -1.2 \times 9.81 = -11.72 \text{ kN/m}^2$

(iii) Pore pressure at C, $u_C = 0$

(iv) Pore pressure at D, $u_D = +\gamma_w \cdot z = 9.81 \times 3 = 29.430 \text{ kN/m}^2$

(c) Effective stress

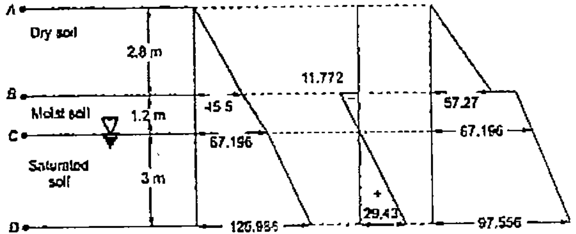
(i) Effective stress at A, $\bar{\sigma}_A = \sigma_A - u_A = 0$

(ii) Effective stress at B, $\bar{\sigma}_B = \sigma_B - u_B = 45.5 - (11.772) = 57.272 \text{ kN/m}^2$

(ii) Effective stress at B, $\bar{\sigma}_B = \sigma_B - u_B = 45.5 - (11.772) = 57.272 \text{ kN/m}^2$

(iii) Effective stress at C, $\bar{\sigma}_C = \sigma_C - u_C = 67.196 - 0 = 67.196 \text{ kN/m}^2$

(iv) Effective stress at D, $\bar{\sigma}_D = \sigma_D - u_D = 126.986 - 29430 = 97.556 \text{ kN/m}^2$

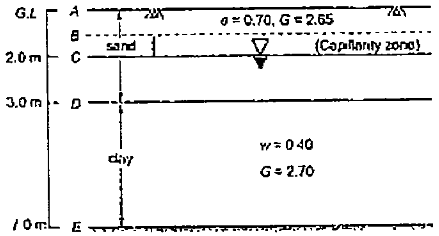


(a) Total stress (σ) (b) Pore pressure (u) (c) Effective stress ($\bar{\sigma}$)

Example 6.8 A soil profile consists of 3 m thick layer of dry sand (G = 2.65, e = 0.7 and effective size = 0.11 mm). A 4 m thick clay layer is present beneath sand (w = 40%, G = 2.70). The ground water table is located at a depth of 2 m from the ground level. Plot variation of total, neutral and effective stress. Assume C = 0.5 cm².

Solution:

The given conditions are shown in figure



Height of capillary rise above water table,

$$h_c = \frac{C}{eD_{10}} = \frac{0.5}{0.70 \times (0.011)} = 65 \text{ cm} = 0.65 \text{ m}$$

Hence sand will be saturated upto 0.65 m above the water table and rest of the sand will completely dry.

$$\gamma_{d,\,sand} = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.70} = 15.30 \text{ kN/m}^3$$

$$\gamma_{sat,\,sand} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.70)\times 9.81}{1+0.70} = 19.33 \text{ kN/m}^3$$

As the clay layer is submerged under water table, clay will be completely saturated.

Using. $S \cdot e = w \cdot G$

$$e = \frac{w \cdot G}{S} = \frac{0.40 \times 2.70}{1} = 1.08$$

$$\therefore \quad \gamma_{sat,\,clay} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.70+1.08)\times 9.81}{1+1.08} = 17.83 \text{ kN/m}^3$$

(a) Total stress (σ)

(i) Total stress at A, $\sigma_A = 0$

(ii) Total stress at B, $\sigma_B = \gamma_{d,\,sand}(2-h_c) = 15.30 \times (2-0.65)$
$= 20.65 \text{ kN/m}^2$

(iii) Total stress at C, $\sigma_C = \gamma_{d,\,sand}(2-h_c) + \gamma_{sat,\,sand} \times h_c$
$= 15.30 \times (2-0.65) + 19.33 \times 0.65$
$= 33.21 \text{ kN/m}^2$

(iv) Total stress at D, $\sigma_D = \gamma_{d,\,sand}(2-h_c) + \gamma_{sat,\,sand}(h_c+1)$
$= 15.30\,(2-0.65) + 19.33\,(0.65+1)$
$= 52.55 \text{ kN/m}^2$

(v) Total stress at E, $\sigma_E = \gamma_{d,\,sand}(2-h_c) + \gamma_{sat,\,sand}(h_c+1) + \gamma_{sat,\,clay} \times 4$
$= 15.30\,(2-0.65) + 19.33\,(0.65+1) + 17.83 \times 4$
$= 123.87 \text{ kN/m}^2$

(b) Pore pressure (u)

(i) Pore pressure at A, $u_A = 0$

(ii) Pore pressure at B, $u_B = -h_c\gamma_w = -0.65 \times 9.81 = -6.38 \text{ kN/m}^2$

(iii) Pore pressure at C, $u_C = 0$

(iv) Pore pressure at D, $u_D = \gamma_w \cdot z = 9.81 \times 1 = 9.81 \text{ kN/m}^2$

(v) Pore pressure at E, $u_E = \gamma_w \times 5 = 9.81 \times 5 = 49.05 \text{ kN/m}^2$

(c) Effective stress ($\bar{\sigma} = \sigma - u$)

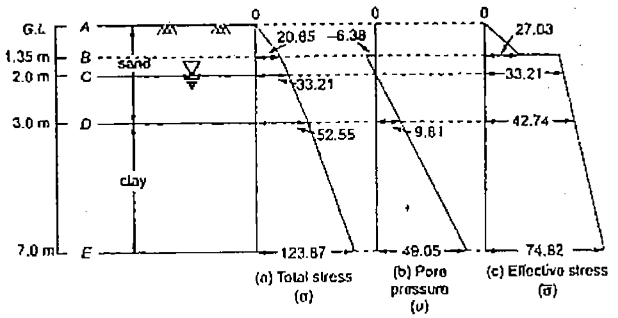
(i) Effective stress at A, $\bar{\sigma}_A = 0$

(ii) Effective stress at B, $\bar{\sigma}_B = 27.03 \text{ kN/m}^2$

(iii) Effective stress at C, $\bar{\sigma}_C = 33.21 \text{ kN/m}^2$

(iv) Effective stress at D, $\bar{\sigma}_D = 42.74 \text{ kN/m}^2$

(v) Effective stress at E, $\bar{\sigma}_E = 74.82 \text{ kN/m}^2$



6.6 Effect of Water Table Fluctuations on Effective Stress

Figure shows a layered soil system consisting of soil-1 and soil-2.

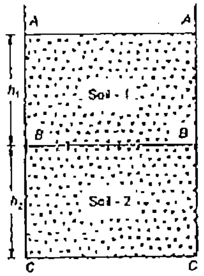


Fig.6.15

Case 1: When water table is below C-C:

(a) Total, Neutral and Effective Stress at *A-A*:

$$\sigma_A = 0$$
$$u_A = 0$$
$$\bar{\sigma} = \sigma_A - u_A = 0$$

(b) Total, Neutral and Effective Stress at *B-B*:

$$\sigma_B = \gamma_{d_1} h_1$$
$$u_B = 0$$
$$\bar{\sigma} = \sigma_B - u_B = \gamma_{d_1} h_1 - 0 = \gamma_{d_1} h_1$$

(c) Total, Neutral and Effective Stress at *C-C*:

$$\sigma_C = \gamma_{d_1} h_1 + \gamma_{d_2} h_2$$
$$u_C = 0$$
$$\bar{\sigma} = \sigma_C - u_C = \left(\gamma_{d_1} h_1 + \gamma_{d_2} h_2\right) - 0 = \gamma_{d_1} h_1 + \gamma_{d_2} h_2$$

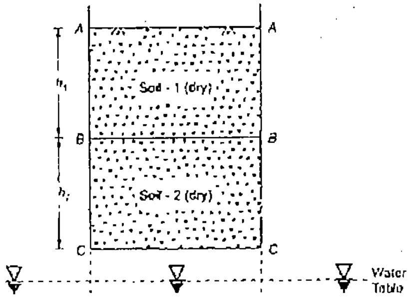


Fig.6.16

Case 2: When water table is at *B-B*:

(a) Total, Neutral and Effective Stress at *A-A*:

$$\sigma_A = 0$$
$$u_A = 0$$
$$\bar{\sigma}_A = \sigma_A - u_A = 0$$

(b) Total, Neutral and Effective Stress at *B-B*:

$$\sigma_B = \gamma_{d_1} h_1$$
$$u_B = 0$$
$$\bar{\sigma}_B = \sigma_B - u_B = \gamma_{d1} \cdot h_1 - 0 = \gamma_{d1} \cdot h_1$$

(c) Total, Neutral and Effective Stress at *C-C*:

$$\sigma_C = \gamma_{d1} \cdot h_1 + \gamma_{(sat)2} \cdot h_2$$
$$u_C = \gamma_w \times h_2$$
$$\bar{\sigma}_C = \sigma_C - u_C = \left(\gamma_{d1} \times h_1 + \gamma_{(sat)2} \times h_2 - \gamma_w \times h_2\right)$$
$$= \gamma_{d1} h_1 + \gamma_{sub} h_2$$

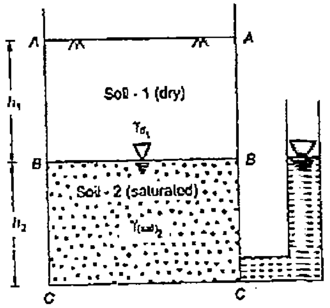


Fig.6.17

Case 3: When Water Table is at Ground Level i.e. *A-A*:

(a) Total, Neutral and Effective Stress at *A - A*:

$$\sigma_A = 0$$
$$u_A = 0$$
$$\bar{\sigma}_A = \sigma_A - u_A = 0$$

(b) Total, Neutral and Effective Stress at *B-B*:

$$\sigma_B = \gamma_{(sat)_1} \cdot h_1$$
$$u_B = \gamma_w \cdot h_1$$
$$\bar{\sigma}_B = \sigma_B - u_B = \gamma_{(sat)_1} \cdot h_1 - \gamma_w \cdot h_1$$
$$= \left[\gamma_{(sat)_1} - \gamma_w\right] h_1$$
$$= \gamma_1' \cdot h_1$$

where, γ_1' = submerged unit weight for soil - 1

(c) Total, Neutral and Effective Stress at *C-C* :

$$\sigma_C = \gamma_{(sat)_1} \cdot h_1 + \gamma_{(sat)_2} \cdot h_2$$
$$u_C = \gamma_w \cdot (h_1 + h_2)$$
$$\bar{\sigma}_C = \sigma_C - u_C = \gamma_{(sat)_1} \cdot h_1 + \gamma_{(sat)_2} \cdot h_2 - \gamma_w (h_1 + h_2)$$
$$= \left[\gamma_{(sat)_1} - \gamma_w\right] \cdot h_1 + \left[\gamma_{(sat)_2} - \gamma_w\right] \cdot h_2$$
$$= \gamma_1' \cdot h_1 + \gamma_2' \cdot h_2$$

where, γ_1' = submerged unit weight for soil - 1

γ_2' = submerged unit weight for soil -2

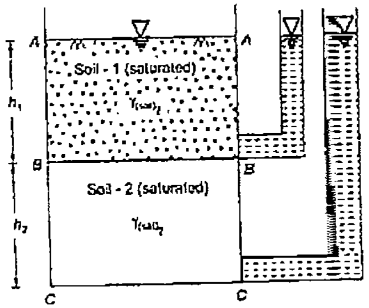


Fig.6.18

Case 4: When Water Table Is Above Ground Level

(a) Total, Neutral and Effective Stress at *A-A*:

σ_A = Overburden pressure due to water column of h_w height

$= \gamma_w \times h_w$

$u_A = \gamma_w \cdot h_w$

$\bar{\sigma}_A = \sigma_A - u_A = \gamma_w h_w - \gamma_w h_w = 0$

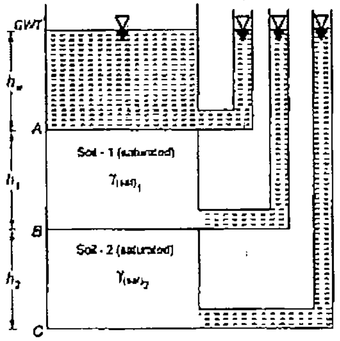


Fig. 6.19

(b) Total, Neutral and Effective stress at *B-B*:

$\sigma_B = \gamma_w \cdot h_w + \gamma_{(sat)1} \cdot h_1 + \gamma_{(sat)2} \cdot h_2$

$u_B = \gamma_w (h_w + h_1)$

$\bar{\sigma}_B = \sigma_B - u_B$

$= \gamma_w \cdot h_w + \gamma_{(sat)1} \cdot h_1 - \gamma_w (h_w + h_1)$

$= (\gamma_{(sat)1} - \gamma_w) \cdot h_1$

$= \gamma'_1 \cdot h_1$

(c) Total, Neutral and Effective stress at *C-C*:

$\sigma_C = \gamma_w \cdot h_w + \gamma_{(sat)1} \cdot h_1 + \gamma_{(sat)2} \cdot h_2$

$u_c = \gamma_w (h_w + h_1 + h_2)$

$\bar{\sigma}_c = \sigma_c - u_c$

$= \gamma_w \cdot h_w + \gamma_{(sat)1} \cdot h_1 + g_{(sat)2} \cdot h_2 - \gamma_w (h_w + h_1 + h_2)$

$= [\gamma_{(sat)1} - \gamma_w] \cdot h_1 + [\gamma_{(sat)2} - \gamma_w] \cdot h_2$

$= \gamma'_1 h_1 + \gamma'_2 h_2$

Where, γ'_1 = submerged unit weight for soil-1

γ'_2 = submerged unit weight for soil-2

6.7 Permeability of Soils

Soil is a particulate material and has pores that provides a passage for water. Such passages vary in size and are interconnected. The permeability of a soil is a property which describes quantitatively, the ease with which water flows through that soil.

Remember

- The same soil may exhibit different degrees of permeability, depending upon its structure. A loose sand is much more permeable than when it is dense.
- A clay soil with a flocculated structure is more permeable than the same soil with a dispersed structure.

6.7.1 Darcy's Law

The flow of free water through soil is governed by Darcy's Law. Darcy established that the flow occurring per unit time is directly proportional to the head causing flow and the area of cross-section of the soil sample but is inversely proportional to the length of the sample. i.e.

$$q \propto \frac{\Delta h}{L} . A$$

$$q = k . \frac{\Delta h}{L} . A = k . i . A$$

Δh = head causing flow

Where, $i = \frac{\Delta h}{L}$, hydraulic gradient

k = Coefficient of permeability of soil

A = Area of cross-section

Note: The Coefficient of permeability of soil (k) is defined as the average velocity of flow which will occur under unit hydraulic gradient. It has the unit same as velocity i.e. cm/sec, or m/day etc.

6.7.2 Discharge Velocity

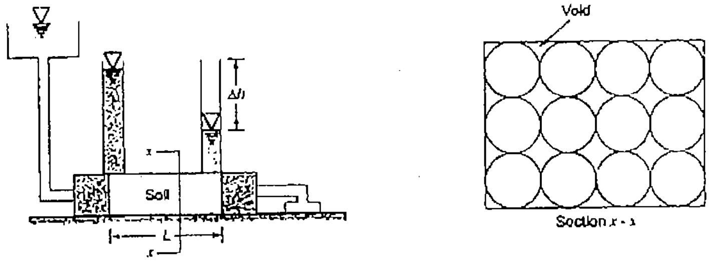


Fig. 6.20

Area of soil specimen at section $X - X = A$

From Darcy's Law

$$q = K . i . A$$

$$\frac{q}{A} = K . i = v$$

Here, *V* is average velocity based on gross area of cross section. The average velocity is also referred as superficial velocity. It is superficial because the actual flow is through pores in the cross section and not through entire cross-sectional area.

6.7.3 Seepage Velocity

The flow through soils, however occurs only through the interconnected pores. The velocity through the pore is called seepage velocity (v_s).

As the flow is continuous, discharge q must be same throughout the system. Thus

$$q = A . v = A_v . c_s$$

$$v = \frac{A_v}{A} . v_S$$

where, A_v = area of voids in total cross-sectional area A

Since, $\frac{A_v}{A} \approx \frac{V_v (\text{Volume of void})}{V (\text{Total volume})} = n$ (n = porosity)

$\therefore$ $v = n \cdot v_s$

i.e., Seepage velocity $v_S = \frac{v}{n}$

Remember: Since $n < 1$, the seepage velocity is always greater than the discharge or superficial velocity.

6.7.4 Limitations of Darcy's Law

- Flow should fulfill continuity conditions.
- The flow should be steady state laminar and one dimensional.
- Soil should be completely saturated.
- No volume changes should occur during or as a result of flow.

6.7.5 Coefficient of Permeability

The coefficient of permeability can be defined using Darcy's equation,

$$q = k \cdot i$$

Hence, the coefficient of permeability may be defined as the velocity of flow which would occur under unit hydraulic gradient. It is also called coefficient of hydraulic conductivity. Coefficient of permeability has dimension of velocity.

Table: Typical value of the coefficient of permeability

S.No.	Soil Type	Coefficient of Permeability (cm/sec)	Drainage Properties
1.	Clean gravel	>1	Very pervious
2.	Sand	1 × 10^{-3} cm/s to 1 cm/s	Pervious
3.	Silt	1 × 10^{-7} cm/s to 1 × 10^{-3} cm/s	Poorly pervious
4.	Clay	< 10^{-7} cm/s	Impervious

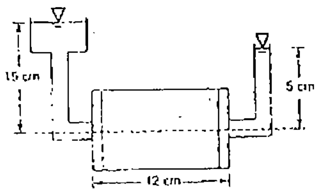
Remember

The coefficient of permeability divided by porosity of soil is known as coefficient of percolation (k_p).

$$k_p = \frac{k}{n}$$

where k_p = Coefficient of percolation
k = Coefficient of permeability
n = Porosity

Example 6.9 For the soil specimen shown in figure, calculate the soil permeability if rate of flow is 2 cc/min. Take area cross-section of soil specimen as 8 cm².



Solution:

Given, L = 12 cm,
A = 8 cm²

Q = 2 cc per minute = $\frac{2}{60}$ cc/sec

$H_L = H_1 - H_2 = 15 - 5 = 10$ cm

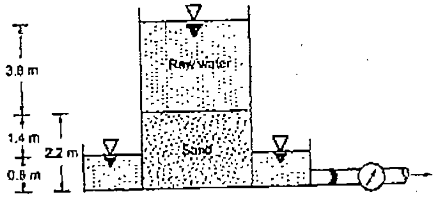
Hydraulic gradient, $i = \frac{H_L}{L} = \frac{10}{12}$

Using Darcy's equation, $Q = k.\ i.\ A$

$$\frac{2}{60} = k.\frac{10}{12}.8$$

$k = 5 \times 10^{-3}$ cm/sec

Example 6.10 A slow sand filter of area 2.85 m² consists 2.2 m sandy layer as shown in figure. Calculate the amount of water supplied per day if permeability of sand is 4.8 × 10^{-5} m/s.



Solution:

Given, A = 2.85 m²
k = 4.8 × 10^{-5} m/s

Let us assume that datum is at bottom of filter.

$\therefore$ Head loss,

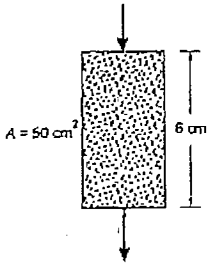
ΔH = Total available at top of sand layer – total head after filtration through sand layer
= (2.2 m + 3.8 m) – (0 + 0.8 m)
= 5.2 m

$\therefore$ Hydraulic gradient, $i = \frac{\Delta H}{\text{thickness of sandlayer}} = \frac{5.2m}{2.2m} = 2.364$

From Darcy's law, $Q = k\ .\ i.\ A$
$= 4.8 \times 10^{-5} \times 2.364 \times 285$
$= 3.234 \times 10^{-1}$ m³/s

$\therefore$ Discharge in a day $= Q \times 24 \times 60 \times 60$ m³/d
$= 3.234 \times 10^{-4} \times 86400$ m³/d
$= 24.94$ m³/d

Example 6.11 Calculate the coefficient of permeability of a soil sample 6 cm in height and 50 cm² in cross sectional area, if a quantity of water equal 450 ml passed down in 10 minutes under an effective constant head of 40 cm. On oven drying, the test specimen weight 495 g. Taking the specific gravity of soil solids as 2.65, calculate the seepage velocity of water during the test.



Solution:

Given,
$$A = 50\,\text{cm}^2$$
$$Q = 450\text{ ml/10 min} = 0.45\text{ l/600 sec}$$
$$= \frac{0.45\times 1000\text{cm}^3}{600\text{sec}} = 0.75\text{ cm}^3\text{/sec}$$
$$i = \frac{\Delta H}{L} = \frac{40\text{cm}}{6\text{cm}} = 6.67$$

From Darcy's Law,
$$Q = k.\, i.\, A$$
$$0.75\text{ cm}^3\text{/s} = k \times 6.67 \times 50\text{ cm}^2$$
$$k = \frac{0.75}{6.67\times 50} = 2.25 \times 10^{-3}\text{ cm/sec}$$

Note that on oven drying, the volume of sample remain the same.

$$\therefore \quad \gamma_d = \frac{W_d}{V} = \frac{495\text{gm}}{(50\times 6)\text{cm}^3} = 1.65\text{ gm/cm}^3$$

Using,
$$\gamma_d = \frac{G.\gamma_w}{1+e},$$
$$\Rightarrow \quad 1.65 = \frac{1\times 2.65}{1+e}$$
$$\therefore \quad e = 0.606$$

Using,
$$n = \frac{e}{1+e},$$
$$\Rightarrow \quad n = \frac{0.606}{1+0.606} = 0.377$$

Now, seepage velocity (v_s) = $\dfrac{\text{Discharge velocity}}{\text{Porosity (n)}}$

where, Discharge velocity (v) = $\dfrac{\text{Discharge}(Q)}{\text{Area of specimen}(A)}$
$$= \frac{0.75cm^3/s}{50cm^2} = 0.015\text{ cm/s}$$

$$\therefore \quad \text{Seepage velocity } (v_s) = \frac{0.015}{0.377}\text{ cm/s}$$
$$= 0.0398\text{ cm/s or }0.398\text{ mm/s}$$

6.8 Permeability of Stratified Soils

In nature, soils are usually deposited in successive layers. Even if the different layers of the deposit are homogeneous within themselves, this may lead to a considerable disparity in the average permeability parallel to the bedding plant and that normal to the bedding planes.

Broadly stratification can be considered as horizontal and continuous, average coefficient for flow in horizontal and vertical directions can be estimated.

6.8.1 Horizontal Flow

Consider the solid profile, shown in figure below consisting of n number of layers with $H_1, H_2, H_3 \ldots H_n$ in thickness with their permeability $k_1, k_2, k_3 \ldots\ldots k_n$ respectively.

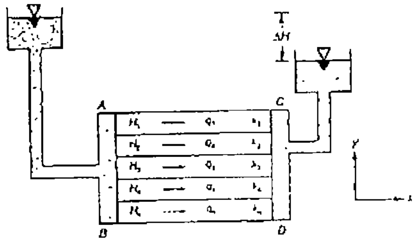


Fig. 6.21

Let H = Total thickness of deposit
$= H_1 + H_2 + H_3 + H_4 \ldots H_n$
k_H = Average permeability in the horizontal direction.

Assume the total head along the line AB to be constant. Similarly, the total head along CD may also be taken as constant but value will be less than that along AB.

$$\therefore \quad i = i_1 = i_2 = i_3 = i_4 \ldots i_n$$

Total discharge through the soil deposit = sum of discharge through the individual layers.

$$\therefore \quad q = q_1 + q_2 + q_3 + q_4 + \ldots q_n$$
$$k_H .\, i .\, A = k_1\, i\, A_1 + k_2\, i\, A_2 + k_3\, i\, A_3 + k_4\, i\, A_4 + \ldots\ldots k_n\, i\, A_n$$
$$\Rightarrow \quad k_H .\, H = k_1 .\, H_1 + k_2 .\, H_2 + k_3 .\, H_3 + k_4 .\, H_4 + \ldots k_n .\, H_n$$
$$k_H = \frac{k_1.H_1 + k_2.H_2 + k_3.H_3 + k_4.H_4 + ..k_n.H_n}{H}$$
$$k_H = \frac{k_1.H_1 + k_2.H_2 + k_3.H_3 + k_4.H_4 + ..k_n.H_n}{H_1 + H_2 + H_3 + H_4 + \ldots.H_n} = \frac{\Sigma kH}{\Sigma H}$$

6.8.2 Vertical Flow

In this case, flow takes place in the direction perpendicular to the stratification.

In this case, to satisfy the continuity condition.

$$q = q_1 = q_2 = q_3 = q_4 = \ldots.\, q_n$$

Let head loss through different layers be $h_1, h_2, h_3, h_4 \ldots.. h_n$

∴ Total head, loss,

$$\Delta H = h_1 + h_2 + h_3 + h_4 + \ldots.h_n \qquad \ldots(i)$$

Also let K_v be the average permeability in vertical direction

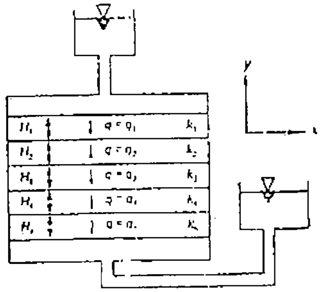


Fig. 6.22

Now,
$$q = k_v .\, i.\, A = k_v.\frac{\Delta H}{H}.A$$

or
$$\Delta H = \frac{q.H}{k_v.A}$$

Similarly,
$$h_1 = \frac{q.H_1}{k_1.A},\quad h_2 = \frac{q.H_2}{k_2.A}.$$

$$h_3 = \frac{qH_3}{k_3.A},\ h_u = \frac{qH_v}{k_u.A}$$

From equation (i), we get

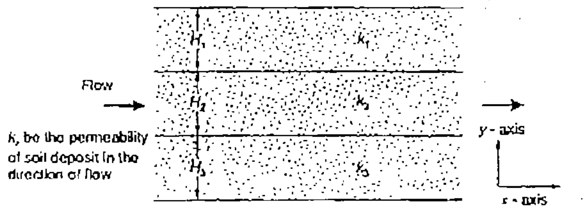
$$\frac{qH}{k_v.A} = \frac{qH_1}{k_1A}+\frac{qH_2}{k_2A}+\frac{qH_3}{k_3.A}+\frac{qH_s}{K_4.A}+\ldots\ldots\frac{qH_n}{k_n}$$

or
$$k_v = \frac{H}{\frac{H_1}{k_1}+\frac{H_2}{k_2}+\frac{H_3}{k_3}+\frac{H_4}{k_4}+\ldots\frac{H_n}{k_n}}$$

Example 6.12 A horizontal stratified soil deposit consists of three layers, each uniform in itself. The permeability of the layer are 8 × 10^{-4}, 50 × 10^{-4} and 15 × 10^{-4} cm/sec; and their thicknesses are 6 m, 3 m and 12 m respectively. Find the effective average permeability of the deposit in horizontal and vertical directions.

Solution:

Since it is a horizontal deposit, the k_x i.e the permeability parallel to bedding planes, will be the horizontal permeability



$$k_x = \frac{k_1H_1+k_2H_2+k_3H_3+\ldots}{H_1+H_2+H_3}$$

Using the above values, we have

$$k_x = k_h = \frac{8\times10^{-1}\times6+50\times10^{-4}\times3+15\times10^{-4}\times12}{6+3+12}$$

$$= \frac{378}{21}\times10^{-4} = 18 \times 10^{-4}\text{ cm/sec}$$

Permeability in vertical direction, k_y, being the permeability of soil deposit in the direction of flow.

$$k_y = \frac{H_1+H_2+H_3}{\frac{H_1}{k_1}+\frac{H_2}{k_2}+\frac{H_3}{k_3}}$$

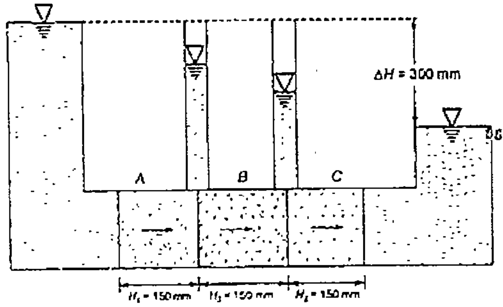
Using the above values, we get

$$k_v = k_v = \frac{6+3+12}{\frac{6}{8\times10^{-4}}+\frac{3}{50\times10^{-4}}+\frac{12}{15\times10^{-4}}}$$

$$= \frac{21\times10^{-4}}{0.75\times0.06+0.80} = \frac{21}{1.61}\times10^{-4}$$

$$= 13.04 \times 10^{-4}\text{ cm/sec}$$

Example 6.13 The soil layers below have a cross-sectional area of 100 mm × 100 mm each. The permeability of each soil is: $k_A = 10^{-2}$ cm/sec; $k_B = 3 \times 10^{-3}$ cm/sec; $k_C = 4.9 \times 10^{-4}$ cm/s. Find the rate of water supply in cm³/hr.



Solution:

Drawing it looks like a horizontal flow, but in reality it is a vertical flow because flow has to cross through every layer. Hence every layer has the same velocity.

$$v = v_1 = v_2 = v_3$$

$$\therefore \quad k_{eq} = \frac{H}{\frac{H_1}{k_1}+\frac{H_2}{k_2}+\frac{H_3}{k_3}} = \frac{450}{\frac{150}{1\times10^{-2}}+\frac{150}{3\times10^{-3}}+\frac{150}{4.9\times10^{-4}}}$$

$$= 1.2 \times 10^{-3}\text{ cm/sec}$$

Head loss during flow, H_L = 300 mm

∴ Hydraulic gradient, $i = \frac{H_L}{H} = \frac{300}{450} = \frac{2}{3}$

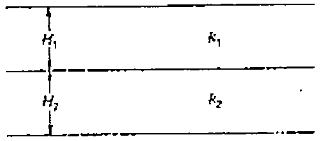
By Darcy's equation, $Q = k_{eq}\,i\,A = 1.2\times10^{-3}\times\frac{2}{3}\times(10\text{cm}\times10\text{cm})\times\frac{3600}{1}\text{ cm}^3/\text{hr}$

$$= 291\text{ cm}^3/\text{hr}$$

Example 6.14 Prove that for stratified deposits of soil, the average permeability in the horizontal direction is greater than the average permeability in the vertical direction.

Solution:

Consider stratified deposits consisting two layers of thicknesses H_1 and H_2 with coefficient of permeability k_1 and k_2 as shown in figure.



We know average permeability in Horizontal and Vertical directions are given by

$$k_H = \frac{k_1 H_1 + k_2 H_2}{H_1 + H_2} = \frac{k_1 H_1 + k_2 H_2}{H}$$

$$k_V = \frac{H_1 + H_2}{\frac{H_1}{k_1} + \frac{H_2}{k_2}} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2}}$$

Therefore, $$\frac{k_H}{k_V} = \frac{(k_1 H_1 + k_2 H_2)/H}{H/[(H_1/k_1) + (H_2/k_2)]}$$

$$= \frac{H_1^2 + H_2^2[((k_1^2 + k_2^2)/k_1 k_2]H_1 H_2}{H^2}$$

$$= \frac{H_1^2 + H_2^2 + [(k_1^2 + k_2^2)/k_1 k_2]H_1 H_2}{H_1^2 + H_1^2 + 2H_1 H_2} \quad [\because H = H_1 + H_2]$$

Since k_1 and k_2 are always positive.

$$(k_1 - k_2)^2 \geq 0$$

or $$k_1^2 + k_2^2 \geq 2\,k_1 k_2$$

$$\frac{k_1^2 + k_2^2}{k_1 k_2} \geq 2$$

The above ratio is equal to 2, only when $k_1 = k_2$ otherwise it is greater than 2. Here if we take different values of k_1 and k_2 then the above ratio will always be greater than 2.

Therefore, $\frac{k_H}{k_V}$ in relation, the numerator is greater than the denominator.

Hence, $k_H > k_V$

6.9 Determination of Coefficient of Permeability

The coefficient of permeability of a soil can be determined using following methods.

(1) Laboratory methods
(2) Field methods
(3) Indirect methods

6.9.1 Laboratory Methods

Laboratory tests can be performed on both undisturbed and remoulded samples. The coefficient of permeability can be determined in laboratory by the following methods.

(a) Constant head permeability test
(b) Variable head permeability test
(c) Capillary permeability test

(a) Constant head permeability test: This test is preferred for coarse grained soils, such as gravels and sands. The complete set up for constant head permeameter is shown in figure below.

An observation is taken by collecting a quantity of water in a graduated jar for a known time.

Let volume of water collected in time 't' is 'V'.

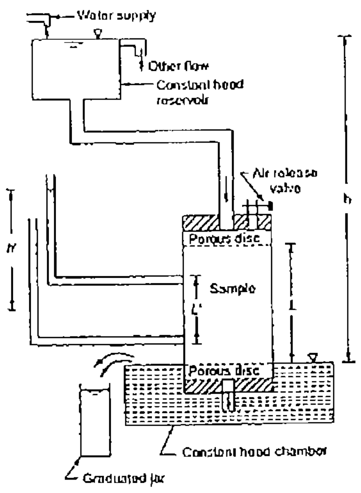


Fig. 6.23 Constant head permeameter

$\therefore$ Discharge, $Q = \frac{V}{t}$

$\Rightarrow \quad k.i.A = \frac{V}{t}$ [By Darcy's Law]

or $\quad k.\frac{h}{L}A = \frac{Q}{t} \quad \left[\because i = \frac{H_L}{L} = \frac{h}{L}\right]$

or $\quad k = \frac{QL}{Ah}$

where, L = distance between manometer taping point
A = Cross sectional area of sample
h = difference in manometric levels

Example 6.15 The following data were recorded in a constant head permeability test:

Internal diameter of permeameter = 7.5 cm
Head lost over a sample length of 18 cm = 24.7 cm
Quantity of water collected in 60 seconds = 626 ml.
Porosity of the soil sample = 44%

Calculate the coefficient of permeability of the soil. Also, determine the discharge velocity and the seepage velocity during the test.

Solution:

Area of soil sample, $A = \frac{\pi}{4}.d^2 = \frac{\pi}{4} \times (7.5)^2 = 44.18\text{ cm}^2$

$H = 24.7\text{ cm},\ L = 18\text{ cm},\ t = 60\text{ sec}$

$\therefore$ Hydraulic gradient, $i = \frac{H_L}{L} = \frac{24.7}{18} = 1.372$

Discharge, $Q = \frac{V}{t} = \frac{626}{60} = 10.433\text{ cm}^3/\text{s}$

From Darcy's equation, $Q = k.i.A$

$$10.433 = k \times 1.372 \times 44.18$$

$$k = 0.172\text{ cm/s}$$

Discharging velocity, $v = \frac{Q}{A} = \frac{10.433}{44.18} = 0.236$ cm/s

Given, porosity of soil sample, $n = 0.44$

$\therefore$ Seepage velocity, $v_s = \frac{v}{n} = \frac{0.236}{0.44} = 0.537$ cm/s

(b) **Variable head permeability test:** Variable head permeability test is used for fine sands and silts which have relatively low permeability. The complete setup for variable head permeability test is shown in figure.

After saturation, the stand pipe of area cross-section 'a' is filled with water and time cross pending to h_1 is noted down. Now water is allowed to fall to h_2, and time t_2 is noted.

Let at any intermediate stage water level be 'h' which falls by 'dh' in time 'dt'. So the head difference is h.

At that stage, let the discharge is 'q'.

$$\therefore \quad q = k.i.A = k.\frac{h}{L}.A$$

Volume of water collected in 'dt',

$$q.dt = a(-dh)$$

$$k.\frac{h}{L}.Adt = -a\,dh$$

Integrating,
$$k.\int_{t_1}^{t_2} dt = \frac{-aL}{A}\int_{h_1}^{h_2}\frac{dh}{h} = \frac{aL}{A}\int_{h_2}^{h_1}\frac{dh}{h}$$

$$k(t_2 - t_1) = \frac{aL}{A}(\log_e h_1 - \log_e h_2)$$

$$= \frac{aL}{A}\log_e\left(\frac{h_1}{h_2}\right)$$

$$k = \frac{aL}{At}\log_e\left(\frac{h_1}{h_2}\right)$$

Where $t = (t_2 - t_1)$

$$k = \frac{2.303aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right)$$

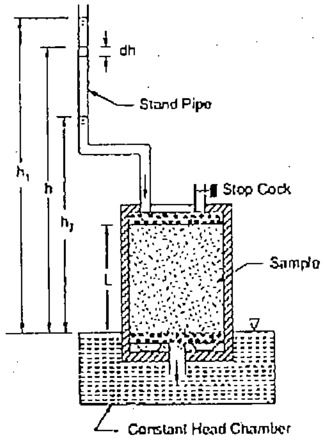


Fig.6.24 Falling head permeameter

NOTE

- Let variable head test is performed in two stages on same soil if, in first stage water level falls from h_1 to h_2 in time interval 't' and during second stage, water level falls from h_2 to h_3 in same time interval 't', then h_1, h_2 and h_3 shall be related as
$$h_2 = \sqrt{h_1 h_3}$$
- The above results can be used to check the consistency of test in a soil. If variable head test is performed in two stages at equal time intervals, then above relation should hold true in each case

Example 6.16 In a falling head permeability test, the head causing fall was initially 90 cm, and it drops 6 cm in 15 minutes. How much time is required for the head to fall to 45 cm.

Solution:

We know, the coefficient of permeability under falling head permeability test is given by

$$k = 2.303\frac{aL}{At}\log_{10}\frac{h_1}{h_2}$$

Here, $h_1 = 90$ cm,

$h_2 = 90 - 6 = 84$ cm

and $t_1 = 15$ minutes

Therefore $t = 2.303\frac{aL}{kA}\log_{10}\frac{h_1}{h_2}$

or
$$\frac{2.303aL}{kA} = \frac{t}{\log_{10}\left(\frac{h_1}{h_2}\right)}$$

$$\Rightarrow \quad \frac{2.303aL}{kA} = \frac{15}{\log_{10}\left(\frac{90}{84}\right)} = 500.61 \text{ minutes} \quad \text{...(i)}$$

Let 'T' be the time interval in which head falls from 90 cm to 45 cm.

Therefore,
$$T = \frac{2.303aL}{kA}\log_{10}\frac{h_1}{h_3} = 500.61\times\log_{10}\frac{90}{45}$$

$$= 150.70 \text{ minutes}$$

Example 6.17 A permeameter of 100 mm diameter with a sample length of 30 cm was used for constant head and falling head test. While conducting a constant head test, the loss of head was 120 cm for the soil sample and rate of flow was 3.2 cm³/s. Find the coefficient of permeability. If a falling head test was performed on the same sample at the same void ratio, find the time taken for the head to fall from 98 cm to 50 cm. The diameter of stand pipe in the falling head test was 25 mm.

Solution:

Constant head test:

$$\text{Area of sample, } A = \pi\left(\frac{100^2}{4}\right) = 7854 \text{ mm}^2$$

$$\text{Hydraulic gradient, } i = \frac{h}{L} = \frac{120}{30} = 4$$

Using, $q = k.\,i.\,A$

Therefore,
$$k = \frac{q}{i.A} = \frac{3.2\times10^3}{4\times7854} = 0.102 \text{ mm/s}$$

Falling head test

Area of stand pipe, $a = \pi\left(\frac{25^2}{4}\right) = 490.9\ \text{mm}^2$

$$\text{Using, } k = 2.303\frac{aL}{At}\log_{10}\frac{h_1}{h_2}$$

or

$$t = \frac{2.303\,aL}{kA}\log_{10}\frac{h_1}{h_2}$$

$$t = \frac{2.303 \times 490.9 \times 300}{0.102 \times 7854}\log_{10}\left(\frac{98}{50}\right)$$

(c) **Capillary permeability test:** It is also known as horizontal capillary test. This test helps us to evaluate the coefficient of permeability (k) as well as value of capillary head (h_c) of the soil sample. The above two test are suitable for saturated soil conditions where as this test can be used for partially saturated soil also.

Procedure:

- In this test, dry or partially saturated soil is placed in a cylinder glass tube of 4 cm dia and 35 cm long. The tube is graduated. Water is allowed to flow from one and other end is open to atmosphere to permit the expulsion of air.

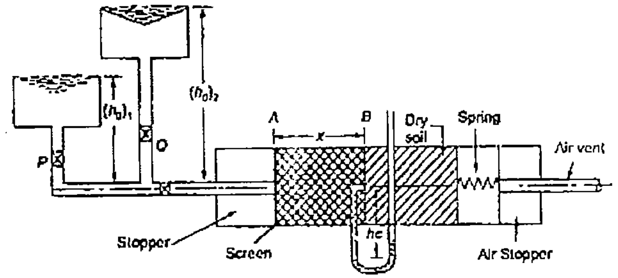


Fig. 6.25

- Let, any time interval 't' after the start of the test, the capillary water travels through a distance x from point A to point B. At point A, there is a pressure head h_0, while at point B, there is a negative pressure h_c.

  ∴ Head lost in flow from A to B,

$$H_L = h_0 - (-h_c)$$
$$= h_0 + h_c$$

∴ Hydraulic gradient,
$$i = \frac{H_L}{L} = \frac{h_0 + h_c}{x}$$

- The seepage velocity through the soil mass (v_s) is given by

$$v_s = \frac{ki}{n} \text{ ... For saturated soil}$$

$$= \frac{ki}{Sn} \text{ ... For partially saturated soil}$$

- Let under head h_0 (constant), water moves distance 'dx' in time 'dt' when it is at a distance x after time 't'.

∴
$$v_s = \frac{dx}{dt} = \frac{ki}{Sn}$$

or
$$\frac{dx}{dt} = \frac{k\left(\frac{h_0 + h_c}{x}\right)}{Sn}$$

or
$$x\,dx = \frac{k}{Sn}(h_0 + h_c).dt$$

Integrating between the limit x_1 and x_2 for x, and corresponding values of time t_1 and t_2, we get

$$\int_{t_1}^{t_2} x\,dx = \left(\frac{h_0 + h_c}{Sn}\right)k\int_{t_1}^{t_2} dt$$

$$\left[\frac{x^2}{2}\right]_{t_1}^{t_2} = \left(\frac{h_0 + h_c}{Sn}\right)k.[t]_{t_1}^{t_2}$$

$$\frac{x_2^2 - x_1^2}{2} = \frac{k(h_0 + h_c)}{Sn}(t_2 - t_1)$$

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{Sn}(h_0 + h_c) \quad \text{...(i)}$$

$S = 100\%$, the above expression becomes

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{n}(h_0 + h_c) \quad \text{...(ii)}$$

where, S = Degree of saturation
h_0 = Head at inlet
h_c = capillary head of soil
n = porosity

- Since k and h_c are the two unknown in above equation. Thus in order to overcome this difficulty, the test is performed in two stages.

Stage-I:

In first stage, soil sample is connected with head h_{0_1}, while value of h_{0_2} is kept closed.

Let under head h_{0_1}, wetted surface of soil moves from x_2 in time t_1 to t_2.

∴
$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{sn}(h_{0_1} + h_c) \quad \text{...(i)}$$

Stage-II:

In second stage, valve of h_{0_1} is closed and soil connected to constant head $h_{0_2}\left(h_{02} > h_{0_1}\right)$ and Let under head h_{0_2} wetted surface of soil moved from x_1' to x_2' in time t_1' to t_2'.

$$\frac{x_2'^2 - x_1'^2}{t_2' - t_1'} = \frac{2k}{Sn}(h_{b_2} + h_c) \quad \text{...(ii)}$$

On solving equation (i) and (ii), values of K and he computed.

Example 6.18 A capillary permeability test was conducted in two stages under a head of 60 cm and 180 cm. In the first stage, the wetted surface moved 1.5 cm to 7 cm in 7 minutes. In the second stage, it advanced from 7 cm to 18.5 cm in 24 minutes. The degree of saturation at the end of the test was 85% and the porosity was 35%. Determine the capillary head and coefficient of permeability.

Solution:

Stage-I	Stage-II
$h_{b_1} = 60$ cm	$x_2' = 18.5$ cm
$h_{b_2} = 180$ cm	$x_1' = 7$ cm
$x_2 = 7$ cm	$t'_2 - t'_1 = 24$ min
$x_1 = 1.5$ cm	$s = 85\%$
$t_2 - t_1 = 7$ min	$n = 35\%$

For stage I,

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{Sn}(h_{b_1} + h_c)$$

$$\frac{7^2 - 1.5^2}{7} = \frac{2k}{0.85 \times 0.35}(60 + h_c)$$

$$k(60 + h_c) = 0.9936 \quad \text{...(i)}$$

Similarly, for stage II,

$$\frac{x_2'^2 - x_1'^2}{t_2' - t_1'} = \frac{2k}{Sn}(h_{b_2} + h_c)$$

$$\frac{18.5^2 - 7^2}{24} = \frac{2k}{0.85 \times 0.35}(180 + h_c)$$

$$\frac{7^2 - 1.5^2}{7} = \frac{2k}{0.85 \times 0.35}(180 + h_c)$$

$$k(180 + h_c) = 1.819 \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\frac{60 + h_c}{180 + h_c} = \frac{0.9936}{1.819}$$

$$1.819(60 + h_c) = 0.9936(180 + h_c)$$

$$109\ 14 + 1.819\,h_c = 178.848 + 0.9936\,h_c$$

$$0.8254\,h_c = 69.708$$

$$h_c = 84.45 \text{ cm}$$

Substituting value of h_c in equation (i), we get

$$k(60 + 84.45) = 0.9936$$

$$144\ 45\,k = 0.9936$$

$$k = 6.87 \times 10^{-3} \text{ cm/min}$$

6.9.2 Field Methods

Laboratory test must be performed, as far as possible, on undisturbed soil samples but obtaining undisturbed sampling is very difficult. The permeability values obtained from the laboratory test may often be quite different from the true value. So for more accurate, representative values, the field tests are conducted.

There are several ways of measuring permeability, in which, the pumping out and pumping in test are quite common. These tests are carried out in boreholes where subsurface explorations are out. These test can be done effectively upto a depth of 30 m and gives the most reliable results.

(a) **Pumping out test:** The pumping out test is more general and accurate method for permeability determination below the water table.

This test is carried out for two basic flow condition: unconfined flow (gravity flow) and confined flow (artesian flow). For confined as well as unconfined flow case, separate formulas have been derived for determining the yield of a well, in terms of drawdown in observation wells, radius of influence, depth of stratum and permeability (k) of soil stratum.

Remember **Aquifer:** An aquifer is defined as a formation of a permeable material which have water holding and water seepage capacity. Examples-sand and gravels

Types of Aquifers:

(I) **Unconfined aquifers:** These are also called non-artesian aquifers. These are the top most water bearing strata having no impermeable overburden lying over them.

(II) **Confined aquifers:** These are also called artesian aquifer. These are sandwiched between two impermeable strata, one at top and other at bottom. In these aquifers, water may be under artesian pressure.

Aquiclude: These are geological formation or stratum of impervious material which do not admit flow of ground water. Example - Clay

Aquitard: These are porous but low permeable. Their seepage capacity is low and discharge under gravity is very low. Example - Silt

Aquifuge: These are a soil mass of rock or an impervious formation which are neither porous nor permeable

Now known the yield and other factors, the permeability k can be easily determined in the field by performing the tests.

(i) Pumping Out Test in Unconfined Aquifer

Specific yield: Specific yield of an unconfined aquifer is the ratio of that volume of water which can flow under gravity to the total volume of soil mass when soil is saturated.

Let V = Total volume of aquifer

V_{wy} = Volume of water that can yielded under gravity

$\therefore$ Specific yield, $S_y = \dfrac{V_{wy}}{V} \times 100$ (in %)

NOTE: Coarse grain soils have more specific yield.

Specific Retention: It is the ratio of that volume of water which can be retained in the voids against the gravity to the total volume aquifer.

$\therefore$ Specific retention, $S_R = \frac{V_{wR}}{V} \times 100$

Note that, $V_{wy} + V_{wR} = V_v$ [Under saturation condition]

or $\frac{V_{wR}}{V} + \frac{V_{wn}}{V} = \frac{V_v}{V}$

or $S_y + S_R = n$ [where n = porosity]

NOTE: Specific retention reduces with increase in particle size.

Analysis of Unconfined Aquifer:

Assumptions:

(i) Darcy's Law is valid and flow is laminar.

(ii) Flow is steady and draw down is stable.

(iii) The test well penetrate through the entire thickness of the aquifer. It means that flow is radial, not spherical i.e. no flow occurs from the base of well.

Two observation wells are made to steady position of water table which must be located within the radius of influence.

Consider an elementary cylinder of soil having radius r, thickness dr and height h. Let the water level falls in the observation well at the rate of dh. Hydraulic gradient,

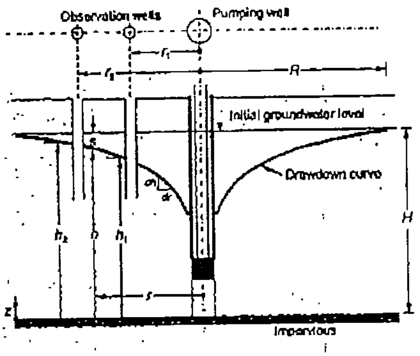


Fig. 6.26 Unconfined Aquifer

$$i = \frac{dh}{dr} \quad \text{and} \quad A = 2\pi rh$$

Discharge into the well = Discharge out of the well i.e pumping rate 'q'

$$\therefore \quad q = k.\ i.\ A = k.\left(\frac{dh}{dr}\right)2\pi rh$$

$$\Rightarrow \quad 2\pi k.\ h.\ dh = q.\frac{dr}{r}$$

Integrating both sides, $2\pi k\int_{h_1}^{h_2} h\,dh = q.\int_{r_1}^{r_2}\frac{dr}{r}$

$$2\pi k\frac{\left[h_2^2 - h_1^2\right]}{2} = q\log_e\left(\frac{r_2}{r_1}\right)$$

$$\therefore \quad k = \frac{2.303q\log_{10}\left(\frac{r_2}{r_1}\right)}{\pi(h_2^2 - h_1^2)} \quad \text{[Thiem's Equation]}$$

NOTE

In Dupit's theory, observation wells are not taken. He took observation at main test well itself. then $h_1 = h_0$, $h_2 = H$, $r_1 = r_0$ (radius of main well), $r_2 = R$ (Radius of Influence) and H = depth of original ground water table from impervious stratum.

$$\text{Thus, } k = \frac{2.303q\log_{10}\left(\frac{R}{r_0}\right)}{\pi(H^2 - H_0^2)} \quad \text{[Dupit's Equation]}$$

Do You Know

Coefficient of transmissibility (T): It is defined as the rate of flow water through a vertical strip of aquifer having unit width extending to full saturation depth under unit hydraulic gradient.

$$\therefore \quad q = k.\ I.\ A$$

$$T = q = k \times 1 \times (1 \times H) = k.H$$

where, H is thickness of aquifer and k is coefficient of permeablity.

Example 6.19 • A well penetrates into an unconfined aquifer having a saturated depth of 100 m. The discharge is 250 lit/min at 12 m drawdown. Assuming equilibrium flow conditions and homogenous aquifer, estimate the discharge at 18 m drawdown.

Solution:

Given, $H = 100$ m, $S_1 = 12$ m

Drawdown, $S_1 = H - h_0$

$\therefore$ $h_0 = 100 - 12 = 88$ m

For first case,

Using, $k = \frac{2.303q\log_{10}\left(\frac{R}{r_0}\right)}{\pi(H^2 - H_0^2)}$

On rearranging,

$$\frac{\pi k}{2.303\log_{10}\left(\frac{R}{r_0}\right)} = \frac{q}{(H^2 - H_0^2)}$$

$$= \frac{250}{100^2 - 88^2} = \frac{250}{2256} \quad \text{...(i)}$$

For second case,

Drawdown, $S_2 = 18$ m

$\therefore$ $H_0 = H - S_2 = 100 - 18 = 82$ m

Again $q = \frac{\pi k(H^7 - H_0^2)}{2.303\log_{10}\left(\frac{R}{r_0}\right)} = \left[\frac{\pi k}{2.303\log_{10}\left(\frac{R}{r_0}\right)}\right] \times (100^2 - 82^2)$

$$= \frac{250}{2256} \times 182 \times 18$$

$$= 363 \text{ litres/minutes}$$

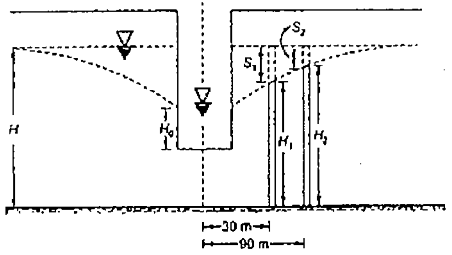
Example 6.20 A 30 cm diameter well penetrates to a depth of 25 m below the static water table. After 24 hours of pumping @ 5400 litres/minutes, the water level in a test well at 90 m is lowered by 0.53 m, and in a well 30 m away, the drawdown is 1.11 m.

(a) What is the transmissibility of the aquifer?

(b) Also determine the draw down in the main well.

Solution:

Since the well penetrates 25 m below the static water table, it evidently is the case of unconfined aquifer.



Given, $H = 25\text{ m}$

Drawdown, $S_2 = 0.53\text{ m}$

$r_0 = 30\text{ m},$

$r_2 = 90\text{ m}$

$S_1 = 1.11\text{ m}$

$h_2 = H - S_2$

$= 25 - 0.53$

$= 24.47\text{ m}$

$h_1 = H - S_1 = 25.11 = 23.89\text{ m}$

$q = 5400\ l/\text{min} = 5.4\text{ m}^3/\text{min} = 0.09\text{ m}^3/\text{s}$

Using Thiem's equation,
$$k = \frac{2.303q\log_{10}\left(\frac{r_2}{r_1}\right)}{\pi(h_2^2 - h_1^2)}$$

Substituting values, we get,
$$k = \frac{2.303 \times 0.09 \times \log\left(\frac{90}{30}\right)}{\pi(24.47^2 - 23.89^2)} = 1.121 \times 10^{-3}\text{ m/s}$$

(a) We know,

Transmissibility, $T = k.H = 1.121 \times 10^{-3} \times 25 = 0.028\text{ m}^2/\text{s}$

(b) To determine the drawdown in the main well, use dupit's equal

$$k = \frac{2.303\log_{10}\left(\frac{r_1}{r_0}\right)}{\pi(H_1^2 - H_0^2)}$$

where, r_0 = radius of main well = 25 cm

H_0 = Steady water level in main well

$$\therefore \quad 1.121 \times 10^{-3} = \frac{2.303\log_{10}\left(\frac{30}{0.15}\right)}{\pi(23.89^2 - H_0^2)}$$

$H_0 = 12.08\text{ m}$

$\therefore$ Drawdown in main well,

$S_0 = H - H_0 = 25 - 12.08 = 12.92\text{ m}$

(ii) Pumping Out Test in Confined aquifer: An artesian well (confined) penetrating the full depth of the aquifer is shown in figure below.

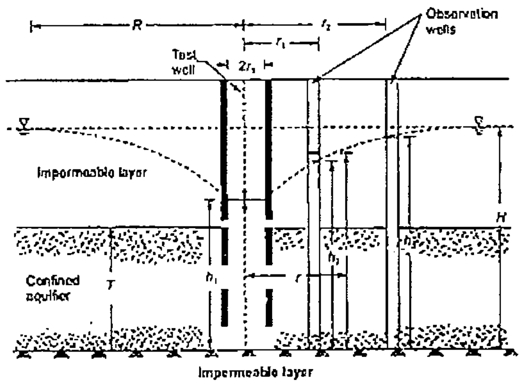


Fig.6.27 Confined Aquifer

In the steady state condition, consider an elementary cylinder of radius *r*, thickness '*dr*' and height *h*.

The rate of flow is given by,

$$q = kiA = k.\frac{dh}{dr}.(2\pi rB)$$

on rearranging and integrating,

$$q.\int_{r_1}^{r_2}\frac{dr}{r} = 2\pi k.B\int_{h_1}^{h_2}.dh$$

$$k = \frac{2.303q\log_{10}\left(\frac{r_2}{r_1}\right)}{2\pi B(h_2 - h_1)} \quad \text{[Thiem's equation]}$$

For Dupit' equation,

$r_1 = r_0$

$h_2 = H$

$H_1 = H_0$

$$\therefore \quad k = \frac{2.303q\log_{10}\left(\frac{R}{r_0}\right)}{2\pi B(H - B)} \quad \text{[Dupit's equation]}$$

Special Case

Spherical Flow In Well: If test well just penetrating over impervious confined stratum. In this case, flow from the base is spherical but very low

The discharge q_s from such a well can, however, be calculated from the equation,

$$Q_s = 2\pi k.\, r_0 (H - H_0)$$
$$= 2\pi K.\, r_0.\, S$$

where,

H_0 = steady water level in well

S = drawdown in well

$= H - H_0$

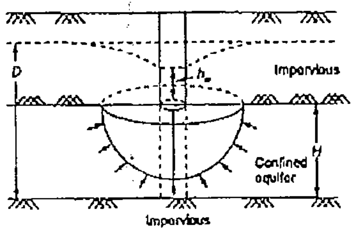
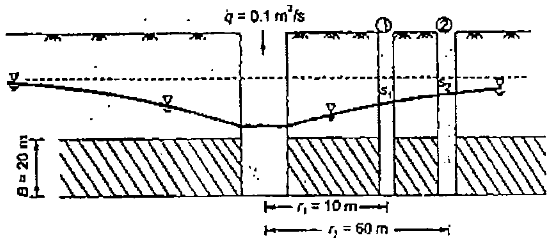


Fig.28

Example 6.21 An aquifer of 20 m average thickness is overlain by impermeable layer of 30 m thickness. A test well of 0.5 m diameter and two observation wells at a distance of 10 m and 60 m the test well are drilled through the aquifer. After pumping at a rate of 0.1 m³/sec for a long time, the following drawdowns were stabilized in these wells; first observation well, 4 m; second observation well, 3 m. Show the arrangement in a diagram. Determine the coefficient of permeability and the drawdown in the test well.

Solution:



Given that $\quad s_1 = 4\text{ m},\ s_2 = 3\text{ m}$

We know that
$$q = \frac{2\pi kB(s_1 - s_2)}{2.303\log_{10}\left(\frac{r_2}{r_1}\right)} \qquad (\because kB = T)$$

$$\Rightarrow \quad q = \frac{2\pi T(s_1 - s_2)}{2.303\log_{10}\left(\frac{r_2}{r_1}\right)}$$

$$\Rightarrow \quad 0.1 = \frac{2\pi T(4-3)}{2.303\log_{10}\left(\frac{60}{10}\right)}$$

$$\Rightarrow \quad T = \frac{2.303 \times 0.1 \times \log_{10} 6}{2\pi}$$

$$\Rightarrow \quad T = 0.0285\ \text{m}^2/\text{sec}$$

$$\therefore \quad k = \frac{T}{B} = \frac{0.0285}{20} = 1.425 \times 10^{-3}\text{m/sec}$$

Again,
$$q = \frac{2\pi T(s_w - s_1)}{2.303\log_{10}\left(\frac{r_1}{r_w}\right)}$$

where s_w is drawdown in well

r_w is radius of well $= \frac{0.5}{2} = 0.25\text{m}$

$$\therefore \quad 0.1 = \frac{2\pi \times 0.0285 \times (s_w - 4)}{2.303\log_{10}\left(\frac{10}{0.25}\right)}$$

$$\Rightarrow \quad s_w = \frac{0.1 \times 2.303 \times \log_{10} 40}{2\pi \times 0.0285} + 4 = 6.06\text{ m}$$

Thus coefficient of permeability of test well, $k = 1.425 \times 10^{-3}$ m/s and drawdown in the test well $s_w = 6.06$ m

(b) Pumping In Test: The pumping in test is suitable for low permeability and thin strata where adequate yield may not be available for pumping out test.

By these test, permeability of soil at the bottom of the bore hole is obtained. Hence these are best suited for permeability determination of stratified deposits.

There are basically two type of pumping-in test:

(i) Open end test

(ii) Packer's Test

(i) Open End Test

- In open-end test, the water flows out of the test hole through its bottom hole.
- An open end pipe is sink into the ground and then soil in the pipe is removed.
- The clean water having temperature greater than underground is added and discharge is measured.
- During test, head is kept constant.
- The coefficient of permeability is determined by the following equation.

$$k = \frac{q}{5.5rH}$$

where, H = difference of levels between the inlet of the causing and the water table.

r = inner radius of casing

q = the constant rate of flow (discharge)

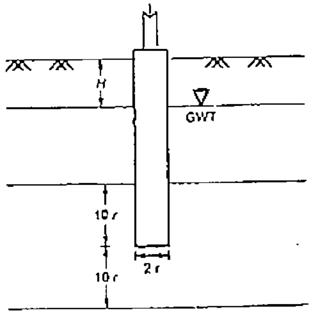


Fig. 6.29

NOTE: For accurate results, the lower end of the pipe should be at a distance of not less than 10r from the top as well as from the bottom of the stratum.

(ii) Packer's Test

- In Packer's test, the water flows out through, the sides of the section of a hole enclosed between packers.

- If L length of pipe is perforated and lower end of the casing is plugged, then, the value of the coefficient of permeability is found by the following equations.

$$k = \frac{q}{2\pi LH}\log_e\left(\frac{L}{r}\right) \quad \text{... if } L \geq 10\ r$$

or $$k = \frac{q}{2\pi LH}\sinh^{-1}\left(\frac{L}{2r}\right) \quad \text{... if } \leq L \leq 10\ r$$

where, r = inner radius of hole

H = difference of water levels at the entry and the ground water table for the hole tested below the water table.

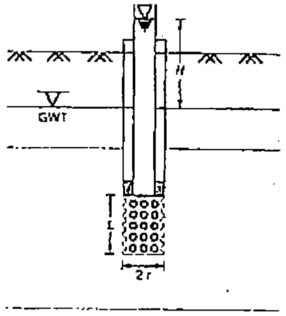


Fig. 6.30

6.9.3 Indirect Methods

(a) Kozney-Carman Equation:

$$\text{Permeability, } k = \frac{1}{k_k}\cdot\frac{\gamma_w}{\mu}\cdot\frac{e^3}{1+e}D_{10}^2$$

k_K = Kozney-Karman constant

e = void ratio

D_{10} = Effective diameter of soil

μ = Dynamic viscosity coefficient

γ_w = Unit weight of water

(b) Allen Hazen's Equation: This equation is valid for granular soils with effective size of 0.1 mm to 3 mm i.e sands

$$k = C.D^2_{10}\,(\text{cm/sec})$$

where, C = Hazen's constant

= 100 to 150

D_{10} = effective size in cm

(c) Loudan's Equation:

$$\log(kS^2) = a + bn$$

where, a and b are constant, their values being 1.365 and 5.15 respectively at 10°C

n = porosity of soil

S = specific surface area

= surface area/volume

Do You Know ? If soil particles are not spherical and are of variable size then, if these particles pass through sieve size 'a' and retain on sieve size 'b' then the specific surface area,

$$S = \frac{6}{\sqrt{ab}}$$

(d) Terzaghi's equation:

$$k = 200\,e^2 D^2_{10}\,(\text{cm/sec})$$

(e) Consolidation equation:

$$k = C_v\gamma_\omega m_v\,(\text{cm/sec})$$

where, γ_ω = unit weight of water (N/cm³)

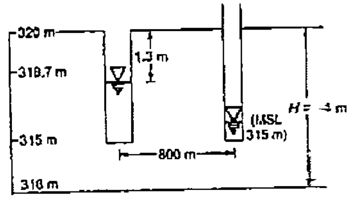
C_v = coefficient of consolidation (cm²/sec)

m_v = Volume compressibility / modulus of volume change (cm²/N)

Example 6.22 A test boring was done at an elevation 320 m MSL, and it was found that the phreatic surface (water table) is 1.3 m below the ground surface. An aquifer was identified and sample of soil was having effective size of 16 mm. A piezometer was installed 800m downstream from the boring and showed water level at elevation 315 m MSL. If thickness of aquifer was 4 m between both points estimate.

(a) The permeability using Hazen's formula (C = 12)

(b) Quantity of flow per m width.



Solution:

Given, $C = 12,\ D = 16\text{ mm}$

(a) Using Allen Hazen formula,

$$k = C(D_{10})^2$$
$$= 12 \times (0.16)^2$$
$$= 0.31\text{ mm/s}$$

(b) The head loss between two points,

$$H_L = (320 - 1.3) - 315 = 3.7\text{ m}$$

∴ Hydraulic gradient, $i = \frac{H_L}{L} = \frac{3.7}{800}$

Using Darcy's Formula,

$$Q = k\,.\,i\,.\,A = 0.31 \times \frac{3.7}{800} \times (4 \times 1) = 1.433 \times 10^{-3}\ \text{m}^3\text{/s}$$

Example 6.23 The coefficient of permeability of fine sand is 0.022 cm/s at a void ratio of 0.72. Estimate permeability of soil if sand is compacted to a void ratio of 0.57. Use Kozeny-Carman formula.

Solution:

From Kozeny-Carman Formula, $$k = \frac{1}{k_k}\frac{\gamma_\omega}{\mu}\frac{e^3}{1+e}D_{10}^2$$

For a given soil the term '$\frac{1}{k_k}\frac{\gamma_\omega}{\mu}D_{10}^2$' would remain constant

Hence, $$k = C\frac{e^3}{1+e}$$

∴ $$\frac{k_1}{k_2} = \frac{\frac{e_1^3}{1+e_1}}{\frac{e_2^3}{1+e_2}}$$

$$\frac{0.022}{k_2} = \frac{(0.72)^3}{1+0.72} \times \frac{1+0.57}{(0.57)^3} = 1.84$$

$$k_2 = \frac{0.022}{1.84} = 0.012 \text{ cm/s}$$

6.10 Factors Affecting Permeability

The permeability depends on the soil properties and fluid properties both.

The Kozney-Carman equation is quite useful, it reflects the effect of factors that affect permeability.

$$k = \frac{1}{k_k} \cdot \frac{\gamma_w}{\mu} \cdot \frac{e^3}{1+e} D_{10}^2$$

(a) Grain Size: The coefficient of permeability (k) includes $D^2{}_{10}$, where D_{10} is a measure of grain size.

i.e., $k \propto D_{10}^2$

If the void ratio is same, then permeability is more in coarse soil than in fine soil.

$$k_{gravel} > k_{sand} > k_{silt} > k_{clay}$$

(b) Void ratio:

Fig.6.31

- As per Kozeny-Carman equation, coefficient of permeability is directly proportional to $\frac{e^3}{1+e}$.

  i.e $k \alpha \frac{e^3}{1+e}$ o $k \alpha e^2$
- If particle size is same, then loose soils are more permeable than dense soils.
- The plot of void ratio (e) against permeability k (at log scale) is approximately a straight line for all soils.

(c) Particle shape:

- It is expressed in terms of specific surface area and permeability relates to specific surface area as $k \alpha \frac{1}{S^2}$
- The angular particles have greater specific surface than the rounded particles.
- For the same void ratio, the soils with angular particles are less permeable than those with rounded particles.

(d) Degree of Saturation: Permeability is directly proportional to degree of saturation i.e $k \alpha$ Degree of saturation (S).

In partially saturated soils, entrapped air causes blockage in the flow of water. Hence, the permeability of partially saturated soil is lesser than that of a fully saturated soil.

(e) Entrapped air and gases: Entrapped air and gases in the voids obstruct flow and cause the reduction of permeability.

As far as possible, sample should be fully saturated before the permeability test.

(f) Structure of Soil particles: For stratified soils, permeability is more in horizontal direction as compared to the vertical direction.

(g) Properties of pore fluid: The coefficient of permeability also includes $\frac{\gamma_w}{\mu}$

i.e $k \alpha \frac{\gamma_w}{\mu}$

For water, relatively the unit weight remain constant but viscosity (μ) decreases with increasing temperature.

i.e $\mu \alpha \frac{1}{T}$

Hence, $k \alpha T$

Do You Know

Usually, permeability is represented at 20°C. test is conducted at T° C, then k_{20} is given by

$$k_{20} = k_T \cdot \frac{\mu_T}{\mu_{20}}$$

$$k_T \cdot \frac{\mu_T}{\mu_{20}} = R_T = \text{Temperature correction factor}$$

$$= 2.42 - 0.475 \log_e(T), \text{ where } T \text{ in}^\circ \text{C}$$

(h) Adsorbed water: Adsorbed water or water film which is strongly attached to the soil solids, reduces flow area available for passage of water, which reduces permeability to some extent.

(i) Impurities: Due to presence of impurities, voids are blocked and the permeability is reduced.

(j) Presence of minerals in water:

- The permeability of clay, depends upon the cation absorb on the surface of mineral.
- If void ratio is constant, the permeability is increased in the following order.

  $k < Na < H < Ca$ for montmorillonite
  $Na < k < Ca < H$ for kaolinite
- For the construction of core of earthen dams, the clay being treated with salt water (Na-water) so that seepage can reduced.

6.11 Coefficient of Absolute Permeability

It was defined by Darcy and this parameter is independent of fluid properties like γ_w and μ. It is only depends on the soil properties.

It is defined as

$$k_0 = k \cdot \frac{\mu}{\gamma_w}$$

Unit: m², cm² or Darcy

$$1 \text{ Darcy} = 0.987 \times 10^{-8} \text{ cm}^2$$

It is also known as intrinsic permeability or specific permeability.

Example 6.24 What is the intrinsic permeability of saturated soil which have hydraulic conductivity of 15.24 m/day. Assuming ground water is at atmospheric pressure at 20°C having density of 998.2 kg/m³ and viscosity of 1.002 × 10⁻³ kg/m-sec.

Solution:

Given,
$\gamma_w = 998.2\ \text{kg/m}^3$
$\mu = 1.002 \times 10^{-3}\ \text{kg/m-s}$

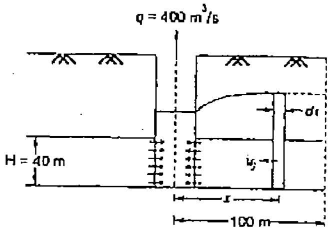
$$k = 15.24\ \text{m/day} = \frac{15.24}{24 \times 60 \times 60} = 1.76 \times 10^{-4}\ \text{m/sec}$$

Using,
$$k_0 = k\frac{\mu}{\gamma_w} = \frac{1.76\times10^{-4}\times1.002\times10^{-3}}{998.2} = 1.77 \times 10^{-4}\ \text{m}^2$$

Example 6.25 A fully penetrating well of radius 0.2 m pumps at a constant rate of 40 m³/hr from a confined sandy aquifer of thickness 40 m and porosity 0.25. Consider only microscopic flow velocity. How long will it take for a pollutant to reach the well if introduced into the aquifer at 100 m from it?

Solution:

Let at any instant of time '*t*', pollutant is at a distance '*x*' from centre of well. Let it moves by *dx* in time *dt* velocity of flow at that time.



$$v_s = \frac{-dx}{dt}$$

Here -ve sign indicate that x is decreasing with time.

$$v_s = \frac{v}{n} = \frac{-dx}{dt}$$

or
$$\frac{(q/A)}{n} = \frac{-dx}{dt}$$

$$\frac{q}{2\pi xH}\times\left(\frac{1}{n}\right) = \frac{-dx}{dt}$$

Microscopic flow means, flow is only through confined layer, Hence flow contribution apart from confined aquifer is neglect

∴
$$\int_0^t dt = \frac{2\pi nH}{q}\int_R^r -x\,dx$$

$$t = \frac{2\pi nH}{q}\left(\frac{R^2 - r^2}{2}\right) = \frac{\pi\times0.25\times40}{400}(100^2 - 0.2^2)$$

$$= 785.395 \text{ hrs or } 32.725 \text{ days}$$

Summary

- Permeability of soil is defined as the property to permit water to percolate through continuously interconnected pores.
- Darcy's Law is valid when flow is laminar, the soil is fully saturated, continuity condition should be valid.
- The apparatus used for the determination of coefficient of permeability of soil in laboratory are called permeameter.

- Constant head permeability test is adopted for coarse-grained soils, where as falling head permeability test is suitable for fine-grained soils.
- Pumping out test is suitable for test below the water table whereas pumping in test can be conducted irrespective of the position of the water table.
- In stratified soils, the average coefficient of permeability in the horizontal direction is more than the average coefficient of permeability in the vertical direction.
- The factors affecting permeability are: Grain size, Void ratio, Particle shape, degree of saturation, entrapped air, structure of soil particle, property of pore fluid, absorbed water etc.
- Absolute permeability of soil is independent of fluid properties, it depends only on soil properties.

Objective Brain Teasers

Q.1 In a falling head permeability test drop in head occurs from 60 m to 40 m in 10 minutes. The cross-sectional area of soil sample and stand pipe is 20 cm² and 2 cm² respectively. If the length of the soil sample is 15 cm then the permeability of soil sample in mm/sec is

(a) 1.01 (b) 0.12
(c) 0.01 (d) 0.001

Q.2 The saturated unit weight of sand in the bed of a pond 20 m deep is 20 kN/m³. Unit weight of water is 10 kN/m³. The effective stress at 4 m below bed level of pond is

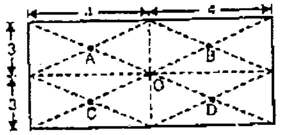
(a) 40 kN/m² (b) 20 kN/m²
(c) 60 kN/m² (d) 80 kN/m²

Q.3 Assertion (A) : Constant head permeability test is not used for fine grained soil.
Reason (R) : Lesser the permeability of soil, lesser is the discharge
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.4 In a falling head permeability test, the time taken for the head to fall from 27 cm to 3 cm is 10 minutes. If the test is repeated with same initial head, i.e. 27 cm, what time would it take for the head to fall 9 cm

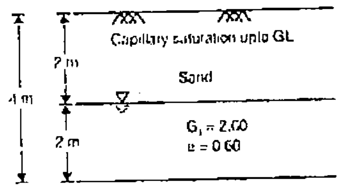
(a) 3 minutes (b) 5 minutes
(c) 6 minutes (d) 7.5 minutes

Q.5 The raft footing of carries a uniform load of 200 kN/m². With the load approximation as shown in the figure, the incremental stress at 5 m below the centre as per Bousinesq's theory is



(a) 13.2 kN/m² (b) 52 kN/m²
(c) 105 kN/m² (d) 26.2 kN/m²

Q.6 The value of effective stress at 4 m depth for the subsoil condition as shown in figure is (Assume γ_w = 10 kN/m³)



(a) 80 kN/m² (b) 20 kN/m²
(c) 60 kN/m² (d) 30 kN/m²

Q.7 A sample of clay and a sample of sand have the same specific gravity and void ratio. Their permeabilities would differ because
(a) their porosities would be different
(b) their degree of saturation would be different
(c) their densities would be different
(d) the size range of their void would be different

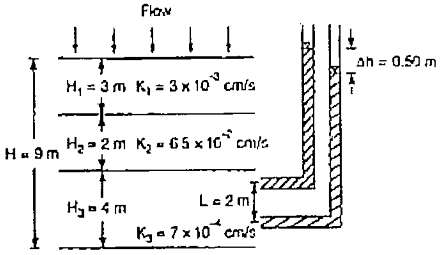
Q.8 Consider the following statements:
1. Soil with high void ratio has always more coefficient of permeability soil with lower void ratio.
2. Constant head permeability test is used for fine grained soil.
3. As temperature increases, the coefficient of permeability of soil also increases.
4. As the specific surface area of soil particle increases the coefficient of permeability decreases.

Which of these statements are correct?
(a) 1, 3 and 4 (b) 2 and 3
(c) 3 and 4 (d) 2, 3 and 4

Q.9 A 30 cm well completely penetrates an unconfined aquifer of depth 40 m. After a long period of pumping at a steady rate of 1500 lpm, the drawdown in two observation wells which are 25 m and 75 m from the pumping well were found to be 3.5 m and 2.0 m respectively. The draw down at the pumping well is
(a) 28.49 m (b) 11.51 m
(c) 10.62 m (d) 12.34 m

Q.10 A capillary permeability test was conducted in two stages under a head of 60 cm and 180 cm respectively at the entry end. In the first stage, the wetted surface moved from 1.5 cm to 7 cm in 7 minutes. In the second stage, it advanced from 7 cm to 18.5 cm in 24 minutes. The degree of saturation at the end of the test was 85% and the porosity was 35%. The capillary head and the coefficient of permeability are respectively.
(a) 84.65 cm and 4.85 × 10^{-3} cm/min
(b) 92.43 cm and 4.85 × 10^{-3} cm/min
(c) 92.43 cm and 6.87 × 10^{-3} cm/min
(d) 84.65 cm and 6.87 × 10^{-3} cm/min

Q.11 Consider the three layered soil strata shown in the figure below. The thickness and coefficient of vertical permeability of each layer is mentioned in the figure.



The total head loss in three layers is
(a) 1.52 m (b) 1.23 m
(c) 1.18 m (d) 0.87 m

Q.12 Specific capacity of a confined well under equilibrium conditions and within the working limits of drawdown is
(a) is constant at all drawdowns.
(b) decreases as the drawdown increases.
(c) increases as the drawdown increases.
(d) None of the above.

Q.13 In a falling head permeability test on a soil, the time taken for the head to fall from h_0 to h_1 is t. The test is repeated with same initial head h_0, the final head h' is noted in time t/2. Which one of the following equation gives the relation between h', h_0 and h_1?
(a) $h' = h_0/h_1$ (b) $h' = \sqrt{h_0/h_1}$
(c) $h' = h_0h_1$ (d) $h' = \sqrt{h_0h_1}$

Q.14 Which one of the following factors are associated with the behaviour of sand mass during earthquake to cause liquefaction?
1. Number of stress cycle
2. The frequency and amplitude of vibration of waves generated by an earthquake
3. Characteristics of sand
4. Relative density

Select the correct answer using the codes given below:
(a) 1, 2, 3 and 4 (b) 1, 2, and 3
(c) 2 and 4 (d) 3 and 4

Q.15 A dry soil with suction capacity '0' rests over a saturated soil. The height of capillary rise is
(a) 0 (b) 1 mm
(c) 1 cm (d) 1 m

Q.16 Consider the following statements:
1. In a sedimentary soil deposit, permeability in the horizontal direction is greater than that in the vertical direction.
2. Permeability of two sand specimens having equal void ratio will be always same

Which of these statement(s) is (are) correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Q.17 Consider the following statements regarding coefficient of permeability 'k' of the soil.
1. It is proportional to square of diameter of soil particle.
2. It decreases with increase in temperature due to reduction in viscosity.
3. It is inversely proportional to square of specific surface.

Which of these statements are correct?
(a) 1 and 3 (b) 2 and 3
(c) 1 and 2 (d) All of these

Q.18 Approximate ratio of the permeabilities of two clay soils, having D_{10} values of 0.8 mm and 0.4 mm respectively, is
(a) 0.25 (b) 2.05
(c) 2 (d) 4

Direction: The following items consists of two statements, one labeled as 'Assertion A' and the other as 'Reason R'. Select your answer to these items using the codes given below.
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

Q.19 Assertion (A): Mohr's circle for unconfined compression test passes through the origin.
Reason (R): In an unconfined compression test, the axial stress is equal to confining stress.

Q.20 Assertion (A): Rankine's earth pressure theory is a simplified form of Coulomb's earth pressure theory.
Reason (R): Coulomb's theory consider effect of pore pressures.

Answers

1. (c)	2. (a)	3. (a)	4. (b)	5. (c)
6. (c)	7. (d)	8. (c)	9. (b)	10. (d)
11. (c)	12. (b)	13. (d)	14. (a)	15. (c)
16. (a)	17. (a)	18. (d)	19. (c)	20. (b)

Hints and Explanations:

1. (c)

$$K = \frac{2.3\,aL}{At}\log_{10}\left(\frac{h_1}{h_2}\right)$$
$$= \frac{2.3 \times 2 \times 150}{20 \times (10 \times 60)}\log_{10}\left(\frac{60}{40}\right)$$
$$= 1.012 \times 10^{-3} \text{ cm/sec}$$
$$= 0.01 \text{ mm/sec}$$

2. (a)

$$\bar{\sigma} = \sigma_T - u$$
$$\sigma_T = (\gamma_w \times 20) + \gamma_{sat} \times 4$$
$$= (10 \times 20) + (20 \times 4) = 280 \text{ kN/m}^2$$
$$u = \gamma_w(20 + 4) = 240 \text{ kN/m}^2$$
$$\bar{\sigma} = 280 - 240 = 40 \text{ kN/m}^2$$

4. (b)

For first test

$$t_1 = \frac{2.303\,aL}{Ak}\log_{10}\left(\frac{h_1}{h_2}\right)$$
$$\Rightarrow \quad 10 = \frac{2.303\,aL}{Ak}\log_{10}\left(\frac{27}{3}\right) \quad ...(i)$$

For second test

$$t_2 = \frac{2.303\,aL}{Ak}\log_{10}\left(\frac{27}{9}\right) \quad ...(ii)$$

From equation (i) and (ii),

$$\frac{t_2}{10} = \frac{\log_{10}\left(\frac{27}{9}\right)}{\log_{10}\left(\frac{27}{3}\right)} = \frac{\log_{10} 3}{\log_{10} 9}$$

$$\Rightarrow \quad \frac{t_2}{10} = \frac{\log_{10} 3}{\log_{10} 3^2} = \frac{\log_{10} 3}{2\log_{10} 3}$$

$$\Rightarrow \quad t_2 = \frac{10}{2} = 5 \text{ minutes}$$

5. (c)

The area for each point = 3 × 4 = 12 m

The load at any point = 12 × 200 = 2400 kN

Due to load at A, the incremental stress below centre O

$$= \frac{3Q\,\tau^3}{2\pi\,R^5} = \frac{3 \times 2400}{2\pi} \times \frac{5^3}{\left(25^2 + 5^2\right)^{5/2}}$$

$$= 26.2 \text{ kN/m}^2$$

Thus the total increment

$$= 4 \times 26.2 = 104.9 \simeq 105 \text{ kN/m}^2$$

6. (c)

$$\gamma_{sat(sand)} = \frac{G+e}{1+e}\gamma_w = \frac{2.6+0.6}{1+0.6} \times 10$$

$$= 20 \text{ kN/m}^3$$

The sand is saturated by gravity flow below water table and by capillary flow upto a height of 2 m above water table.

$$\therefore \quad \sigma_{at\,4\,m} = \gamma_{sat} \times 4 = 20 \times 4 = 80 \text{ kN/m}^2$$

$$U = 2 \quad \gamma_w = 20 \text{ kN/m}^2$$

$$\therefore \quad \bar{\sigma} = 80 - 20 = 60 \text{ kN/m}^2$$

9. (b)

$$q = 1500 \text{ litres/min}$$

$$= 1.5 \text{ m}^3\text{/min} = 0.025 \text{ m}^3\text{/sec}$$

$$h_2 = H - s_2 - 40 - 2 = 38 \text{ m;}$$

$$h_1 = H - s_1 = 40 - 3.5 = 36.5 \text{ m}$$

We know that,

$$q = \frac{1.36k(h_2^2 - h_1^2)}{\log_{10} r_2/r_1}$$

$$\text{or} \quad 0.025 = \frac{1.36k(38^2 - 36.5^2)}{\log_{10} 75/25}$$

From which,

$$k = 7.848 \times 10^{-5} \text{ m/sec}$$

At the well face,

$$q = \frac{1.36k(h_1^2 - h_w^2)}{\log_{10}(r_1/r_w)}$$

$$\therefore \quad 0.025 = \frac{1.36 \times 7.848 \times 10^{-5}(36.5^2 - h_w^2)}{\log_{10}\left(\frac{25}{0.15}\right)}$$

$$\text{or} \quad h_w^2 = 36.5^2 - 520.42 \simeq 811.83$$

$$h_w = 28.49 \text{ m}$$

∴ Draw down at well face,

$$s_w = H - h_w = 40 - 28.49 = 11.51 \text{ m}$$

10. (d)

First stage:

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k}{Sn} \times (h_{01} + h_c)$$

$$\frac{7^2 - 1.5^2}{7} = \frac{2k}{0.85 \times 0.35}(60 + h_c)$$

$$\text{or} \quad k(60 + h_c) = 0.9934 \qquad \text{...(i)}$$

Second stage:

$$\frac{18.5^2 - 7^2}{24} = \frac{2k}{0.85 \times 0.35}(180 + h_c)$$

$$\text{or } k\,(180 + h_c) = 1.8175 \qquad \text{...(ii)}$$

From equation (i) and (ii)

$$\frac{180 + h_c}{60 + h_c} = \frac{1.8175}{0.9934} = 1.8296$$

from which $h_c = 84.65$ cm

Hence from equation (i),

$$k = 6.87 \times 10^{-3} \text{ cm/min.}$$

11. (c)

$$K_v = \frac{H}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3}}$$

$$= \frac{9}{\frac{3}{3 \times 10^{-3}} + \frac{2}{6.5 \times 10^{-2}} + \frac{4}{7 \times 10^{-4}}}$$

$$= 1.33 \times 10^{-3} \text{ cm/s}$$

For continuity of flow, velocity is the same

$$K_3 \cdot \frac{\Delta h}{L} = K_v \cdot \frac{\Delta h_{total}}{H}.$$

where Δh_{total} = total head loss in three layers

$$\therefore \Delta h_{total} = K_3 \cdot \frac{\Delta h}{L} \cdot \frac{H}{K_v}$$

$$= 7 \times 10^{-4} \times \frac{0.50}{2} \times \frac{9}{1.33 \times 10^{-3}}$$

$$= 1.184 \text{ m}$$

12. (b)

$$\text{Specific capacity} = \frac{1}{C_1 + C_2 Q}$$

(C_1 and C_2 are constant)

This equation shows that specific capacity decreases as Q increases and Q increases only when drawdown is increased by heavier pumping and hence, specific capacity decreases as drawdown increases.

13. (d)

In falling head permeability test

$$k = 2.3\,\frac{aL}{At}\log_{10}\left(\frac{h_0}{h_1}\right)$$

$$= 2.3\frac{aL}{A(t/2)}\log_{10}\left(\frac{h_0}{h'}\right)$$

$$\therefore \log_{10}\left(\frac{h_0}{h_1}\right) = 2\log\left(\frac{h_0}{h'}\right)$$

$$\Rightarrow \quad \frac{h_0}{h_1} = \left(\frac{h_0}{h'}\right)^2$$

$$\Rightarrow \quad h' = \sqrt{h_0 h_1}$$

14. (a)

Liquefaction of soil during earthquake depends on stress cycle, type of soil characteristics, vibration and relative density of sand.

15. (c)

Suction capacity

$$= \log_{10} h_c = 0$$

$$\Rightarrow \quad h_c = 1 \text{ cm}$$

17. (a)

1. $k \propto (\text{size})^2$

$$\therefore \quad k = CD_{10}^2$$

2. $k \propto$ temperature

3. $k \propto$ specific surface

4. $k \propto \left(\frac{1}{\text{viscosity}}\right)$

$$k = \frac{1}{c} \times \frac{1}{s^2} \times \frac{\gamma_w}{\mu} \times \frac{e^3}{1+e}$$

18. (d)

$$k_1 = CD_{10}^2$$

$$\therefore \quad \frac{k_1}{k_2} = \left(\frac{D_{1_{10}}}{D_{2_{10}}}\right)^2$$

$$\frac{k_1}{k_2} = \left(\frac{0.8}{0.4}\right)^2$$

$$\Rightarrow \quad \frac{k_1}{k_2} = 4.0$$

19. (c)

The all round pressure (confining pressure) is zero in unconfined compression test and the Mohr's circle passes through origin.

Student's Assignments

Q 1. A sand sample of 25 cm length was subjected to a constant head permeability test in a permeameter having an area of 30 m². A discharge of 100 cm³ was obtained in a period of 1 minute under a head of 39 cm. Mass of dry sand in the sample was 1350 g. The specific gravity of the sand particles was 2.67. Determine:

(i) the coefficient of permeability

(ii) the superficial velocity and

(iii) the seepage velocity

[Ans. 0.0356 cm/s, 0.056 cm/s, 0.0183 c m/s]

Q 2. Calculate the coefficient of permeability of a soil sample 8 cm in height and cross sectional area 60 cm^2. It is observed that in 12 minutes, 600 ml of water passed down under as effective constant head of 50 cm. On oven drying, the test specimen weight 750 gm. Taking $G_s = 2.70$, calculate the seepage velocity of water during the test.

[Ans. 2.22 × 10^{-3} cm/sec, 0.33 cm/sec.]

Q 3. A falling head permeability test was performed on a sand sample and the following data were recorded:

Cross-sectional area of permeameter = 100 cm^2; length of the soil sample = 15 cm; area of the stand pipe = 1cm^2; time taken for the head to fall from 150 cm to 50 cm = 8 min, temperature of water was 25°C. Dry mass of soil specimen = 2.2 kg and $G_S = 2.68$. Calculate the coefficient of permeability of the soil for a void ratio of 0.70 and standard temperature 20°C.

[Ans. 2 × 10^{-4} cm/s]

Q 4. The following data were recorded in a constant head permeability test:

Head lost over a sample length of 18 cm = 24.7 cm

Quantity of water collected in 60 seconds = 626 ml

Internal diameter of permeability = 7.5 cm

Porosity of the soil sample = 44%

Calculate the coefficient of permeability of the soil. Also determine the discharge velocity and the seepage velocity during the test.

[Ans. [0.172 cm,s 0.236 cm/s, 0.537 cm/s]

Q 5. In a capillary permeability test conducted in two stages under a head of 50 cm and 200 cm, in the first stage the wetted surface rose from 20 mm to 80 mm in 6 minutes. In the second stage it rose from 80 mm to 200 mm in 20 minutes. If the degree of saturation is 90% and the porosity is 30%, determine the capillary head and the coefficient of permeability.

[Ans. 170.49 cm, 1.02 × 10^4 cm/sec]

Q 6. A sand deposit of 12 m thickness overlies a clay layer. The water table is 3 m below the ground surface. In a pump out test, the water is pumped out at rate of 540 litres/min when steady-state conditions are reached. Two observation wells are located at 18 m and 36 m from the centre of the test well. The depth of the drawdown curve are 1.8 m and 1.5 m respectively, for these two wells. Determine the coefficient of permeability.

[Ans. 17.96 cm/min]

Q 7. A pumping out test was carried out in the field, in order to determine the average coefficient of permeability of 18 m thick sand layer. The ground water table is located at a depth of 2.2 m below the ground level. A steady state was reached when the discharge from the well was 21.5 lit/sec. At this stage, the drawdown in the test well was 2.54 m, while the drawdown in two observation well situated at 8 m and 20 m from the test well were found to be 1.76 m and 1.27 m respectively.

Determine:

(i) Coefficient of permeability of the sand layer in m/day

(ii) Radius of influence of test well

(iii) Effective size of the sand.

[Ans. 38.70 m/day, 161.29 m, 0.212 mm]

Q 8. A soil has the coefficient of permeability of 0.4 × 10^{-4} cm/sec at a void ratio of 0.65 and temperature of 30° C. Determine the coefficient of permeability at the same void ratio and a temperature of 20° C. At 20°C, $\gamma_w = 0.998$ g/ml and $\mu = 0.0101$ poise and at 30°C, $\gamma_w = 0.996$ gm/ml and $\mu = 0.008$ poise. What would be the coefficient of permeability at a void ratio of 0.75 and temperature of 20° C?

[Ans. 0.317 × 10^{-4} cm/sec;
0.422 × 10^{-4} cm/sec]

■■■■

CHAPTER 07

Seepage Through Soils

7.1 Introduction

Seepage is the flow of water under gravitational forces in a permeable medium. Flow of water takes place from a point of high head to a point of low head. In one dimensional flow, where Darcy's law is valid, the velocity of flow is taken constant at every point over a cross section normal to the direction of flow. However many practical situations (like flow through earth dams, sheet piles) the flow of water is not unidirectional and velocity of water does not remain constant. In such cases, the quantity of seepage and other parameters such as hydraulic gradient and pore water pressure are estimated by the help of flow nets. The concept and construction of flow nets are based on Laplace's equation of continuity.

7.2 Type of Head

There are three types of head available in fluid flow:

(i) Velocity head (ii) Pressure head (iii) Datum or elevation head

(i) Velocity head

- It is equal to $\frac{v^2}{2g}$.
- Since laminar flow occurs during the seepage and velocity is very small during laminar flow. Hence in seepage analysis, velocity head may be neglected.

(ii) Pressure head

- It is equal to $\frac{P}{\gamma_w}$.
- If a piezometer or an open stand pipe is inserted at a point of flow, water would stand at a particular height inside the piezometer.
- The actual height of rise of water column in the piezometer, represents the pressure head.

(iii) Datum or elevation head

- It is represented by 'Z'.
- The elevation or datum head at a point is the vertical distance of that point measured from an assumed datum. Generally, datum is assumed at tail water level.

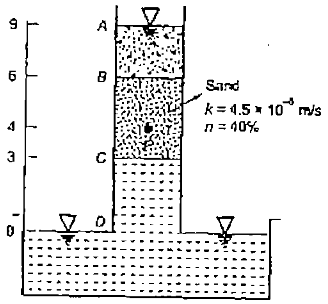
7.3 Total Head

- Total head = velocity head + pressure head + elevation head
- Here, Total head = $0 + \frac{P}{\gamma_w} + z$
- If we insert a stand pipe at a point of flow, the elevation of water level in stand pipe with reference to the datum is equal to total head.
- In seepage analysis, velocity head is neglected. Hence total head and piezometric head both are equal.

7.3.1 Head Loss

- The difference in total head between two points in a soil through which flow is occurring is represented by the head loss during the flow between these points.
- If flow is occurring through the soil sample, the total head is assumed to reduce during flow.

Example 7.1 Determine the pressure head, elevation head and total head at the entering end, exit end and point P of the sample. What are the discharge (superficial) and seepage velocity of flow?



Solution:

Assume datum is taken at $EL\ 0(m)$.

Elevation of point (m)	Elevation Head (m)	Pressure Head (m)	Total Head E.H + P.H (m)	Head Loss (m)
A	9	0	9	0
B	6	3	9	0
C	3	–3	0	9
D	0	0	0	9
P	4	–1	3	6

It is convenient to first determine the total head loss due to sand specimen which is equal to the difference of initial and final water level. Now determine the elevation and total heads, and then calculate the pressure head by substracting elevation head from the total head. This procedure can be adopted for any point within the soil.

Head loss due to sand specimen, $\Delta H = 9 - 0 = 9\text{m}$

For point P, the head loss upto P is calculated proportionately as $\frac{9}{3} \times 2 = 6$ m. Thus the total head at P is equal to total head at B (9 m) minus the head loss upto P (6 m).

i.e. $9 - 6 = 3$ m

The pressure head is computed last; it is the total head – elevation head

i.e. $3 - 4 = -1$ m

We know, discharge (superficial) velocity is given by

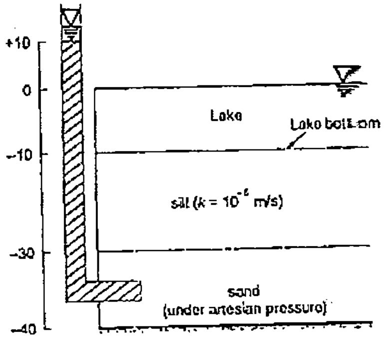
$$v = k.i = k.\frac{\Delta H}{3} = 4.5 \times 10^{-5} \times \frac{9}{3}$$

$$= 1.35 \times 10^{-4} \text{ m/s or } 0.135 \text{ mm/sec}$$

The seepage velocity, v_s is given by the equation,

$$v_s = \frac{v}{n} = \frac{0.135}{0.4} = 0.3375 \text{ mm/sec}$$

Example 7.2 The soil profile below a lake with water level at elevation = 0 m and lake bottom at elevation = 10 m is shown in the figure. A piezometer (stand pipe) installed in the same layer shows a reading of +10 elevation. Assume that the piezometric head is uniform in the sand layer. The quantity of water in cum per day, flowing into the lake from the sand layer through the silt layer per unit area of the lake bed would be?



Solution:

Given, $k = 10^{-6}$ m/s

$= 10^{-6} \times 60 \times 60 \times 24$ m/day

$= 0.0864$ m/day

$L = 20$ m,

$A = 1\ \text{m}^2$

Total head available at the bottom of silt stratum

= Elevation head + Pressure head

= 10 + 40 = 50 m

Total head available at the top of silt stratum

= Elevation head + Pressure head

= 30 + 10 = 40 m

∴ Head loss due to silt stratum,

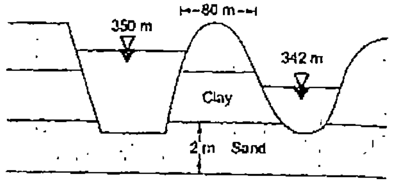
$\Delta H = 50 - 40 = 10$ m

Hydraulic gradient, $i = \frac{\Delta H}{L} = \frac{10}{20} = 0.5$

By Darcy's law $Q = k.i.A$

$= 0.0864 \times 0.5 \times 1 \text{ cum/day} = 4.32 \times 10^{-2} \text{ cum/day}$

Example 7.3 A canal and a river run parallel with an average distance of 80 m apart. The elevation of the water surface in the canal is at 350 m and in the river at 342 m. A stratum of sand intersect both the river and the canal below their water levels. The sand is 2 m thick and is sandwiched between strata of impervious clay. Compute the seepage loss from the canal in m³/day-km, if the permeability of sand is 4.5×10^{-5} m/s.



Solution:

Head loss, $H_L = 350 - 342 = 8$ m

∴ Hydraulic gradient, $i = \frac{H_L}{L} = \frac{8}{80} = \frac{1}{10}$

Using Darcy's equation, $Q = k.i.A$

$$= 4.5 \times 10^{-5}\left(\frac{86,400\,s}{\text{day}}\right) \times \frac{1}{10} \times (2\text{ m} \times 1000\text{ m})$$

$$= 777.6\text{ m}^3/\text{day-km}$$

7.4 Seepage Pressure and its Effect on Effective Stress

-

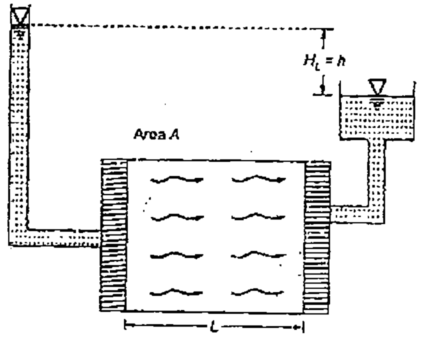


Fig. 7.1 Seepage pressure in Soil

- **Seepage Pressure:** When water flows through the saturated soil mass, it exerts the pressure over the solids by the virtue of viscous friction and is referred as seepage pressure.

  Hence, seepage pressure is the pressure exerted by the water over the soil solids in the mass through which it percolates.
- If '*h*' is the hydraulic head (i.e Head loss) under which flow is taking place, then seepage is given by

$$p_s = h\gamma_w$$

Also, $$p_s = \frac{h}{z} z\gamma_w$$

or $$p_s = iz\gamma_w$$

If A be the area cross-section, then seepage force is given by

$$P_s = \text{seepage pressure } (p_s) \times A$$

$$= iz\gamma_w \times A$$
$$= i.(z \times A).\gamma_w$$
$$= i.V.\gamma_w$$

Seepage per unit volume is known as "specific seepage force" which is given by

$$P_{ss} = \frac{P_s}{V} = \frac{i.V.\gamma_w}{V} = i\gamma_w$$

NOTE Seepage pressure always acts in the direction of flow. Hence vertical pressure (effective stress) at any given section in flow condition, may either increase or decrease.

∴ $$\bar{\sigma}' = \bar{\sigma} \pm P_s$$

where $\bar{\sigma}$ = effective stress under no flow condition

P_s = seepage pressure under flow condition

7.4.1 No Flow Condition

- The figure shows a tank filled with submerged soil. Since the valve at the base is closed, no seepage will occur.

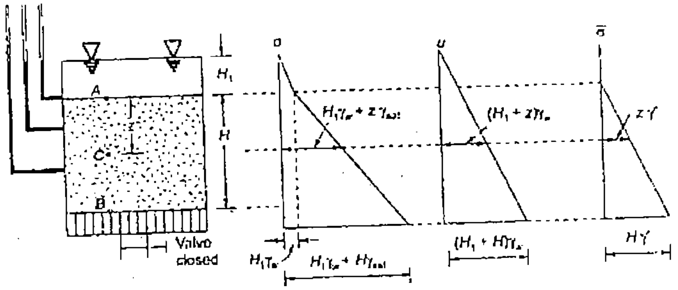


Fig. 7.2

- Pressure head, datum head and total head are tabulated below.

Point	A	B	C
Pressure head	H_1	$H_1 + H$	$H_1 + Z$
Datum head	H	0	$H - Z$
Total head	$H_1 + H$	$H_1 + H$	$H_1 + H$

- Total stress, pore water pressure and effective stress and tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w(H_1 + H)$	$\gamma_w(H_1 + Z)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$\gamma_{sat} H - \gamma_w H$ $= \gamma' H$	$\gamma_{sat} Z - \gamma_w Z$ $= \gamma' Z$

The variations of σ, u and $\bar{\sigma}$, are shown in figure above.

7.4.2 Downward Flow

- Figure shows a tank filled with submerged soil, since the valve at the base is open and downward seepage is allowed. The downward seepage increases in effective stress.

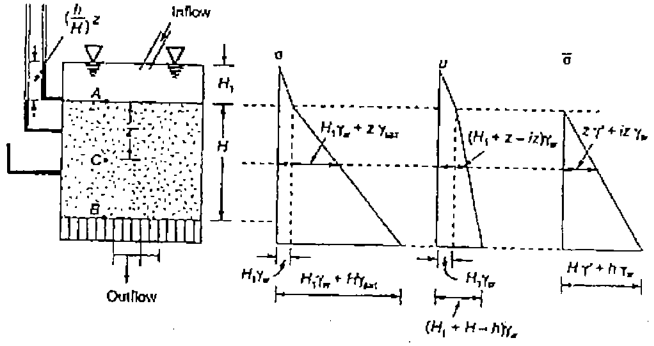


Fig.7.3 Variation of σ, u and $\bar{\sigma}$ with depth downward flow

- Pressure head, datum head and total head are tabulated below:

Point	A	B	C
Pressure head	H_1	$H_1 + H - h$	$H_1 + Z - iZ$
Datum head	H	0	$H - Z$
Total head	$H_1 + H$	$H_1 + H - h$	$H_1 + H - iZ$

where, h = hydraulic head (head loss) under which flow takes place from A to B

Hydraulic gradient, $i = \frac{h}{H}$

- The total vertical stress at any point in soil mass is due submerged weight of soil mass and seepage pressure depending upon the direction of flow.
- Total stress, pore water pressure and effective stress are tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$	$\gamma_{sat} Z + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w (H_1 + H - h)$	$\gamma_w (H_1 + Z - iZ)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$= (\gamma_{sat} - \gamma_w) H + \gamma_w h$ $= \gamma' H + \gamma_w h$	$= (\gamma_{sat} - \gamma_w) Z + \gamma_w iZ$ $= \gamma' Z + \gamma_w iZ$

The variations of σ, u and $\bar{\sigma}$ are shown in figure above

7.4.3 Upward Flow

- Figure shows, valve at the bottom of tank is open and upward seepage is allowed.

Point	A	B	C
Pressure head	H_1	$H_1 + H + h$	$H_1 + Z + iZ$
Datum head	H	0	$H - Z$
Total head	$H_1 + H$	$H_1 + H + h$	$H_1 + H + iZ$

Where, h = hydraulic head (head loss) under which flow takes place from A to B

Hydraulic gradient, $i = \frac{h}{H}$

Total stress, pore water pressure and effective stresses are tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$	$\gamma_{sat} Z + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w (H_1 + H + h)$	$\gamma_w (H_1 + Z - iZ)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$= (\gamma_{sat} - \gamma_w) H - \gamma_w h$ $= \gamma H - \gamma_w h$ $= \gamma' H - iZ\gamma_w$	$= (\gamma_{sat} - \gamma_w) H - \gamma_w h$ $= \gamma H - \gamma_w h$ $= \gamma H - iZ\gamma_w$

The variations of σ, u and $\bar{\sigma}$ are shown in figure.

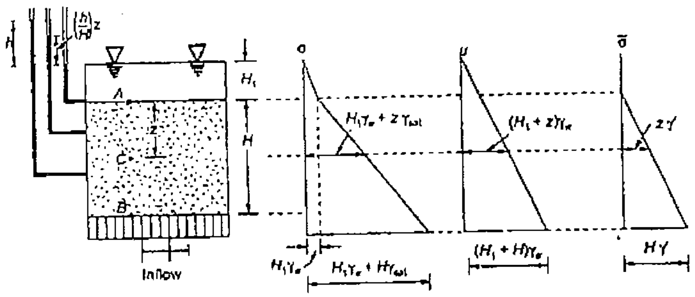


Fig. 7.4 Variation of σ, u and $\bar{\sigma}$ with depth for upward flow

7.5 Quick Sand Condition

For upward flow condition, effective stress at any point within soil mass is given by

$$\bar{\sigma} = \bar{\sigma} - p_s$$
$$= \gamma' z - p_s$$

It is clear from above equation, upward seepage pressure decreases effective stresses in the soil mass.

- If the seepage pressure is such that it equals the submerged weight of the soil mass, then effective stresses at that location reduces to zero. Under such condition, cohesionless soil mass loses all shear strength. Now soil mass has a tendency to move also with the flowing water in the upward direction.

- This process in which soil particles are lifted over the soil mass is called quick sand condition. It is also known as 'boiling of sand' as the surface of sand looks it is boiling.
- At quick sand condition, net effective stress is reduced to zero. i.e.

$$\bar{\sigma} = 0$$

$$\therefore \quad \gamma' z = p_s = 0 \qquad \text{[where } p_s = \text{seepage pressure]}$$

$$\text{or} \quad \gamma' z - iz\gamma_w = 0$$

$$i = \frac{\gamma'}{\gamma_w} = i_{cr}$$

- The hydraulic gradient under which quick sand condition occurs is termed as critical hydraulic gradient. If void ratio and specific gravity of soil is known, then i_{cr} may be given as

$$i_{cr} = \frac{\gamma'}{\gamma_w} = \frac{\frac{(G-1)\gamma_w}{1+e}}{\gamma_w}$$

$$\therefore \quad i_{cr} = \frac{G-1}{1+e}$$

- For fine sand and silts for which specific gravity ≈ 2.65 and void radio $e \simeq 0.65$,

$$i_{cr} = \frac{2.65-1}{1+0.65} \simeq 1$$

- At quick sand condition, cohesionless particles of fine sand may start flowing with the water which may result in piping failure below the hydraulic structure.
- In order to prevent quick sand or piping failure, the hydraulic gradient should be less then critical hydraulic gradient. Hence factor of safety against quick sand failure or piping failure is

$$\text{F.O.S.} = \frac{i_c}{i} \qquad \text{where,} \quad i = \frac{h_l}{L}$$

NOTE

- Quick sand is not a type of sand. It is a flow condition which exists in cohesionless soil mass where effective stresses are reduced to zero in upward flow conditions.
- Quick sand conditions is found only in fine sand and coarse silts and it is not being observed in the case of gravels, coarse sands and clays.
- In cohesive soils which posses inherent cohesion, shear strength is not reduced to zero even when effective stresses are reduced to zero.
  For cohesive soils,

$$\tau_f = c' + \bar{\sigma}\tan\phi'$$

$$\text{If} \quad p_s = \gamma' z$$

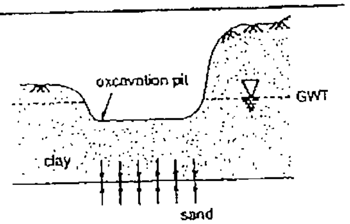
$$\text{then,} \quad \bar{\sigma} = \gamma' z - p_s = 0$$

$$\text{or} \quad \tau_f = c' + 0$$

- Quick sand condition is not observed in gravels and coarse sands which are highly permeable soils. As per Darcy's law a large discharge is required to generate critical hydraulic gradient.

$$Q = k.i.A; \qquad i = \frac{Q}{k.A}$$

Remember Quick sand condition is generally observed when excavation is done below the GWT and water is pumped out to keep the excavated area free from it or it is observed when sand is under artesian pressure and overlain by a clay layer.



Example 7.4 A soil sample (sand) obtained from trench has water content of 38%. The specific gravity of solid is 2.65. Determine the critical hydraulic gradient for the soil. Also determine the maximum permissible upward gradient, if a factor of safety of 3 is considered.

Solution:

Given, $w = 38\%$, $G = 2.65$

For saturated soil,

$$1 \times e = wG$$

$$\Rightarrow \quad e = 0.38 \times 2.65 = 1.007$$

Now, critical hydraulic gradient,

$$i_{cr} = \frac{G-1}{1+e}$$

$$\therefore \quad i_{cr} = \frac{2.65-1}{1+1.007} = 0.822$$

If the factor of safety is 3, then

Maximum permissible upward gradient, $i = \frac{i_{cr}}{F.O.S} = \frac{0.822}{3} = 0.274$

Example 7.5 A 3 m thick soil stratum has coefficient of permeability 3×10^{-7} m/sec. A separate test have porosity 40% and bulk unit weight 21 kN/m³ at a moisture content of 31%. Determine the head at which upward seepage will cause quick sand condition. What is the flow required to maintain critical conditions?

Solution:

Given $k = 3 \times 10^{-7}$ m/s, $n = 0.40$

$\gamma = 21$ kN/m³, $w = 0.31$

Using,

$$n = \frac{e}{1+e}$$

$$0.40 = \frac{e}{1+e},$$

$$\therefore \quad e = 0.667$$

Using,

$$\gamma_d = \frac{\gamma}{1+w},$$

We get

$$\gamma_d = \frac{21}{1+0.31} = 16.03 \text{ kN/m}^3$$

Also, $\gamma_d = \dfrac{G.\gamma_w}{1+e}$

$\therefore$ $16.03 = \dfrac{G\times 9.81}{1+0.667}$

or $G = \dfrac{16.03\times 1.667}{9.81} = 2.724$

Now, we can use, $i_{cr} = \dfrac{G-1}{1+e} = \dfrac{2.724-1}{1+0.667} = 1.034$

Also, $i_{cr} = \dfrac{H_l}{L}$

$\therefore$ $\dfrac{H_L}{3} = 1.034$

or $H_L = 1.034 \times 3 = 3.1$ m

Hence, 3.1 m of head is sufficient to cause quick sand condition.

The flow required to cause quick sand condition can be obtain by Darcy's equation.

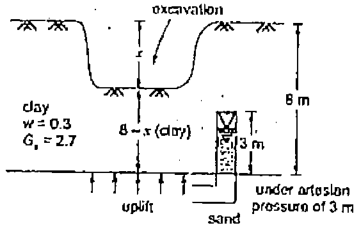
$Q = k.i.A$

$= 3\times 10^{-7} \times 1.034 \times 1$ (Assume unit area)

$= 3.1 \times 10^{-7}$ m³/sec/m²

Example 7.6 At a given location, 8 m thick saturated clay; natural water content = 30%, G_S = 2.7 is under-lain by sand. The sand layer is under artesian pressure equivalent to 3 m of water head. It is proposed to make an open excavation in the clay. How deep can this excavation be made before the bottom heaves?

Solution:



Let the safe depth of excavation = x m

$\therefore$ Thickness of clay overlaying sand at excavation site $= (8-x)$ m

The bottom of excavation will remain stable as long as the downward stresses due to clay stratum are more than upward pressure due to artesian head in sand.

For maximum depth of excavation, the downward and upward pressure should be just equal

$\therefore$ $(8-x)\gamma_{sat.clay} = 3\times\gamma_w$ ...(i)

where $\gamma_{sat\,clay} = \dfrac{G+e}{1+e}\gamma_w$

where $e = wG = 0.3 \times 2.7 = 0.81$

$\therefore$ $\gamma_{sat.clay} = \dfrac{(2.7+0.81)}{1+0.81}\times 9.81 = 19.02$ kN/m³

On substituting values of $\gamma_{sat\,clay}$ and γ_w, we get

$(8-x)\times 19.02 = 3\times 9.81$

$8-x = 1.55$

$x = 8-1.55 = 6.45$ m

$\therefore$ Maximum depth of cut $= 6.45$ m

Alternative approach:

Head loss during flow, $H_L = 3-(8-x)$

$\therefore$ Hydraulic gradient, $i = \dfrac{3-(8-x)}{(8-x)}$

For maximum depth of cut, $i = i_{cr}$

$\therefore$ $\dfrac{3-(8-x)}{(8-x)} = \dfrac{G-1}{1+e}$

$\dfrac{3-(8-x)}{(8-x)} = \dfrac{2.7-7}{1+0.81} = 0.939$

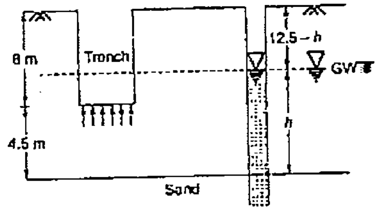
$x-5 = 7.514-0.939x$

$1.939x = 12.514$

$x = 6.45$

Example 7.7 A layer of clay of thickness 12.5 m is underlain by sand. The saturated unit weight of the clay is 18.5 kN/m³. When the depth of an open trench excavated in the clay reached a depth of 8 m, the bottom cracked and the water started entering the trench from below. Determine the height to which water would have risen from the top of sand in a bore hole it were drilled into sand prior to the excavation. Take γ_w = 10 kN/m³

Solution:



Let h be the height of the water table above the top of the sand as shown in figure.

We know quick sand condition will start in sand when net effective stress become zero at top of sand.

Total stress on top of sand below trench,

$\sigma = 4.5\,\gamma_{sat}$

Let h_w be the artesian pressure head

$\therefore$Pore pressure due to artesian head,

$u = h_w\gamma_w = h_w \times 10$

$\therefore$ Effective stress, $\bar{\sigma} = \sigma - u$

$= 4.5\,\gamma_{sat} - 10\,h_w$

For quick sand condition, $\bar{\sigma} = 4.5\,\gamma_{sat} - 10\,h_w = 0$

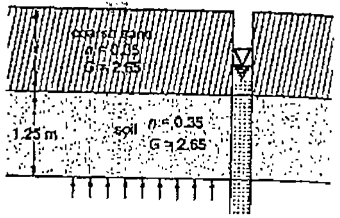
$h_w = \dfrac{4.5\gamma_{sat}}{10} = \dfrac{4.5\times 18.5}{10} = 8.325$ m

Example 7.8 A 1.25 m layer of soil having the porosity 0.35 and specific gravity 2.65 is subjected to an upward seepage head of 1.85 m. What depth of coarse sand would be required above the existing soil to provide a factor of safety of 2 against piping? Assume that coarse sand has the same porosity and specific gravity as the soil and there is negligible head loss in sand.

Solution:

Let x be the depth of coarse sand required above existing soil.

Using, $n = \dfrac{e}{1+e} = 0.35$



$0.35 + 0.35\,e = e$

$e = 0.5385$

Now, $\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$

$= \left(\frac{2.65+0.5385}{1+0.5385}\right)\times 9.81$

$= 20.33\ \text{kN/m}^3$

$\therefore$ $\gamma_{sub} = \gamma_{sat} - \gamma_w$

$= 20.33 - 9.81$

$= 10.52\ \text{kN/m}^3$

$$\text{F.O.S} = \frac{\text{Downward force}}{\text{Upward force}}$$

or $$\frac{\text{Downward force due to wt of soil}}{\text{Upward force due to 1.85 m seepage head}} = 2$$

$\therefore$ $$\frac{(1.25+x)\times\gamma_{sub}}{1.85\times\gamma_w} = 2$$

$$\frac{(1.25+x)\times 10.52}{1.85\times 9.81} = 2$$

$(1.25 + x) = 3.45$

$x = 2.20\ \text{m}$

7.6 Laplace Equations

- When flow takes place in 2-D then Darcy's equation cannot be used. In such case of seepage, Laplace equations are used which represents the loss of energy head in any resistive medium like soil.
- For isotropic medium

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2} = 0$$

- A soil is said to be non isotropic when it has different permeability in two-mutually perpendicular direction. For non-isotropic medium, Laplace equation becomes

$$k_x\frac{\partial^2 H}{\partial x^2}+k_y\frac{\partial^2 H}{\partial y^2} = 0$$

where $\frac{\partial H}{\partial x}$ = loss of energy head in x-direction

$\frac{\partial H}{\partial y}$ = loss of energy head in y-direction

- The solution of two Laplace equations for the potential and flow functions takes the form of two families of orthogonal curves.

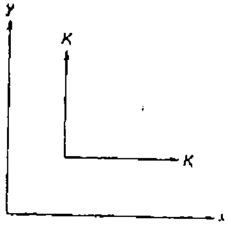


Fig. 7.5 Isotropic medium

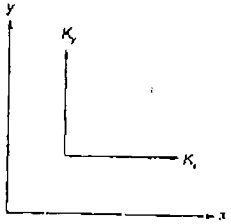


Fig. 7.6 Non-isotropic medium

- One set of curve (ψ-lines) represent the trajectories of seepage and are termed as flow lines. The space between two adjacent flow lines are known as flow channel.
- The another set of curve (ϕ-lines) represents lines of equal head and are termed equipotential lines. The head loss caused by water crossing two adjacent equipotential lines is known as potential drop.

Assumptions made in Laplace Equations:

- Soil is homogeneous and fully saturated.
- Flow is Laminar and Darcy's Law is valid
- Pore water and soil solids are assumed to be incompressible.

7.7 Flow Nets

The entire pattern of flow lines and equipotential lines is referred as a flow net. It is the solution of Laplace's equation for relevant boundary conditions. Thus, a flow net is a graphical representation of the head and direction of seepage at every point.

7.7.1 Properties of Flow Nets

- Flow lines (ψ-lines) and equipotential (ϕ-lines) meet each other othogonally.
- Flow lines and equipotential lines are smooth continuous curves, being either elliptical or parabolic in shape.
- There can be no flow across the flow line and velocity of flow is always perpendicular to equipotential lines.
- Area bounded between two adjacent flow lines is called flow channel or flow path and quantity of water flowing through each channel is same.
- Area bounded between two adjacent equipotential lines and adjacent flow lines is referred to a flow field.
- For isotropic medium, flow field is approximately square which may either be linear or curvilinear.
- For non-isotropic medium, flow field is rectangular which may either be linear or curvilinear.
- Loss of head between two equipotential lines i.e equipotential drop remains constant for all the equipotential lines.
- Flow net remain unchanged if boundary conditions are not altered i.e flow net remain same for the same set of boundary conditions.

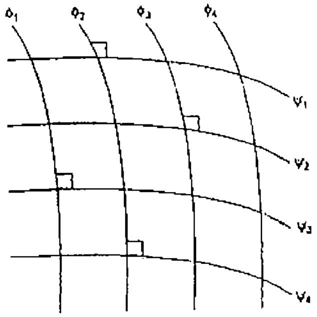


Fig. 7.7

7.7.2 Application of Flow Nets

Flow net can be used for the determination of seepage, seepage pressure, hydrostatic pressure and exit gradient.

1. **Determination of Seepage**

Let Δ*q* be the discharge through each flow channel under the total head of *H*.

(a) For Isotropic Medium:

- For isotropic medium, flow fields are square in shape. Let '$b \times b$' be the size of flow field

$\therefore$ $$\Delta q = k.i.A = k.\frac{\Delta h}{l}(b \times 1)$$

But for isotropic medium, $l = b$

$\therefore$ $$\Delta q = k.\Delta h \qquad \text{...(i)}$$

If 'H' be the total head loss and N_d be the number of potential drops.

then, $$\Delta h = \frac{H}{N_d}$$

Therefore, equation (i) becomes

$$\Delta q = k.\frac{H}{N_d}$$

Let N_f be the total number of flow channels.

The total discharge through the complete flow net per unit length is given as

$$q = \Delta q \times N_f = k.\frac{H}{N_d}.N_f$$

$$= kH.\frac{N_f}{N_d}$$

- The ratio $\frac{N_f}{N_d}$ is independent of k and H and is characteristic of the flow net. This is called shape factor of the flow net.

(b) For Non-Isotropic Medium:

- In this case the equation of continuity assumes the form

$$k_x\frac{\partial^2 H}{\partial x^2} + k_y\frac{\partial^2 H}{\partial y^2} = 0 \qquad \text{...(i)}$$

- Solution of above equation cannot be given as it is not in Laplacian form, so we have to transform it by substituting a transformed coordinate x_T for x such that

$$x_T = x\sqrt{\frac{k_y}{k_x}}$$

or $$x = x_T\sqrt{\frac{k_x}{k_y}}$$

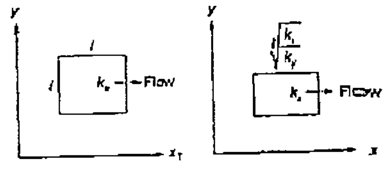
$$\therefore\ k_x\frac{\partial^2 H}{\partial^2_{x_T}\left(\sqrt{\frac{k_x}{k_y}}\right)} + k_y\frac{\partial^2 H}{\partial y^2} = 0$$

$$k_x.\frac{\partial^2 H}{\partial x_T^2}.\frac{k_y}{k_x} + k_y\frac{\partial^2 H}{\partial y^2} = 0$$

$$\frac{\partial^2 H}{\partial x_T^2} + \frac{\partial^2 H}{\partial y^2} = 0 \qquad \text{...(ii)}$$

Equation (ii) is in standard laplacian form, flow net (flow field) for which can be drawn for transformed section. On this transformed scale, flow field will be a square

Let, equivalent permeability on transformed scale be k_o. Also let Δ_{q_T} and Δ_{q_A} be the quantity of flow through transformed and actual section respectively.



(i) Transformed scale (ii) Actual scale

Fig.7.8

For transformed section, $$\Delta_{q_T} = k_o\frac{\Delta h}{l}(l \times 1)$$

For actual section, $$\Delta_{q_A} = k_x.\frac{\Delta h}{l\sqrt{\frac{k_x}{k_y}}}(l \times 1)$$

But $$\Delta_{q_T} = \Delta_{q_A}$$

Therefore, $$k_o.\frac{\Delta h}{l}\times l = k_x.\frac{\Delta h}{l\sqrt{\frac{k_x}{k_y}}}.l$$

$$k_o = \sqrt{k_x k_y}$$

Thus seepage quantity, $$\Delta q = k_o H\frac{N_f}{N_d}$$

Note: If flow is in 3-D, $k_o = \sqrt[3]{k_x.k_y.k_z}$

Example 7.9 A concrete gravity dam, 120 m long and 75 m wide, lies on a permeable soil with a coefficient of permeability of 40 × 10^{-3} mm/s. The head of water is maintained at 32 m upstream and zero at the tail-end. The soil is under lain by an impervious stratum. The depth from the base of the dam to impervious stratum is 38 m. A flow net constructed for this condition yielded 8 flow channel and 18 equipotential drops. What is the seepage loss per day under the dam, considering a two-dimensional flow.

Solution:

Given,

Coefficient of permeability, $k = 40 \times 10^{-3}$ mm/s

$$= \frac{40\times10^{-3}}{1000}\ \text{m/s}$$

Head loss, $H = 32$ m

No. of flow channels, $N_f = 8$

No. of flow channel, $N_d = 18$

For homogenous isotropic soil, $$q = kH.\frac{N_f}{N_d} = \frac{40\times10^{-3}}{1000}\times32\times\frac{8}{18}\ \text{m}^3\text{/s/m}$$

$$= \frac{40\times10^{-3}}{1000}\times32\times\frac{8}{18}\times(60\times60\times24)\ \text{m}^3\text{/d/m}$$

$$= 49.152\ \text{m}^3\text{/d/m}$$

Example 7.10 An earth dam is built on an impervious foundation with a horizontal filter at the base near the toe. The permeability of the soil in the horizontal and vertical directions are 3.2×10^{-2} mm/sec and 1.5×10^{-2} mm/sec respectively. The full reservoir level is 30 m above the filter. A flow net constructed for the transformed section of the dam, consist of 5 flow channels and 13 potential drops. Estimate the seepage loss per m length level of the dam.

Solution:

Given:

Horizontal permeability, $k_H = 3.2 \times 10^{-2}$ mm/s

Vertical permeability, $k_V = 1.5 \times 10^{-2}$ mm/s

$\therefore$ Equivalent permeability, $k_e = \sqrt{k_H.k_V} = \sqrt{3.2\times10^{-2}\times1.5\times10^{-2}}$

$= 2.19 \times 10^{-2}$ mm/s

Available Head above filter, $H = 30$ m

No. of flow channels, $N_f = 5$

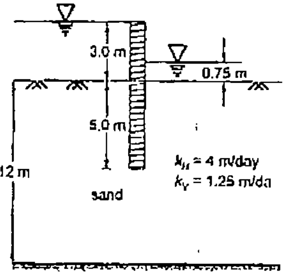
No. of potential drop, $N_d = 13$

$\therefore$ Seepage per unit length of dam, $q = k_e.H.\dfrac{N_f}{N_d} = \dfrac{2.19\times10^{-2}}{1000}\times30\times\dfrac{5}{13}$

$= 2.527 \times 10^{-4}$ m³/sec/m or 0.2527 l/sec

Example 7.11 A sand deposit 12 m thick overlie on impervious soil. A vertical sheet pile penetrate 5 m into the sand deposit. The water level on U/S and D/S is 3.0 m and 0.75 m above the ground surface respectively. The horizontal and vertical permeability for sand are 4 m/day and 1.25 m/day respectively. A flow net construction reveals that there are 12 flow channels and 26 potential drops. Determine the seepage flow per day.

Solution:



Equivalent permeability, $k_e = \sqrt{k_H.k_V}$

$= \sqrt{4\times1.25} = 2.236$ m/day

Head loss, $H = 3.0 - 0.75 = 2.25$ m

No. of flow channels, $N_f = 12$

No. of potential drops, $N_d = 26$

$\therefore$ Seepage per day per unit length of sheet pile,

$$q = k_e.H.\frac{N_f}{N_d}$$

$$= 2.236\times2.25\times\frac{12}{26}$$

$$= 3.77 \text{ m}^3\text{/day}$$

2. Determination of Seepage Pressure

- Let n_d be the number of potential drop (each of value Δh) by a water particle before reaching a point where seepage pressure is required. Let H be total head governing flow, hence net available head at point under consideration will be,

$$h_1 = H - n_d\Delta h$$

Hence seepage pressure,

$$p_s = h_1\gamma_w$$

or $$p_s = (H - n_d\Delta H)\gamma_w$$

- This pressure acts in the direction of flow.

3. Determination of Uplift Pressure

- Hydrostatic pressure at any point after 'n_d' equipotential drop is given by

$$u = h_w\gamma_w$$

where h_w is piezometric head.

$$h_w = h_1 \pm z \text{ or } (H - n_d\Delta H) \pm 2$$

where z is elevation head

- Generally the downstream water level is usually considered as the datum and all points above the datum are considered as positive.

4. Determination of Exit gradient

Exit gradient is the hydraulic gradient at the D/S end of the flow line where the percolating water leaves the soil mass and emerges out as free water.

$$i_e = \frac{\Delta h}{l}$$

where l is length of the smallest square in the last flow field and Δh is potential drop.

7.8 Flow Through Non-Homogeneous Section

- If there is a change in soil conditions, the flow lines are deflected at the interface of the soil with varying permeability, k_1 and k_2.
- Let the potential drop from P to Q and from R to S be ΔH, then

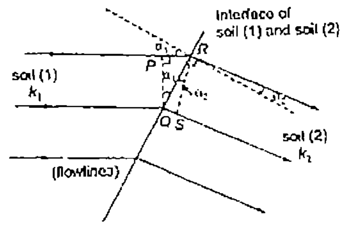


Fig. 7.9 Flow through non-homogenous section

$$\Delta q = k_1\left(\frac{\Delta H}{PR}\right)PQ$$

$$= k_2\left(\frac{\Delta H}{QS}\right)RS$$

But $$\tan\alpha_1 = \frac{PR}{PQ}$$

and $$\alpha_2 = \frac{QS}{RS}$$

$\therefore$ $$\frac{k_1}{\tan\alpha_1} = \frac{k_2}{\tan\alpha_2}$$

or $$\frac{k_1}{k_2} = \frac{\tan\alpha_1}{\tan\alpha_2}$$

- If $k_1 > k_2$, than $\alpha_1 > \alpha_2$, flow get deflected towards normal.
- If $k_1 < k_2$, than $\alpha_1 < \alpha_2$, flow get deflected away from normal.